Geodätische Woche 2015
7-9 September 2015
Stuttgart

Application of the Rosborough formulation to analysis of the gravitational gradients of the GOCE mission

Nico Sneeuw [1,2], Khosro Ghobadi-Far [2], Mohammad A. Sharifi [2]

[1] Institute of Geodesy, University of Stuttgart; sneeuw@gis.uni-stuttgart.de

[2] School of Surveying and Geospatial Engineering, University of Tehran; k.ghobadi@alumni.ut.ac.ir; sharifi@ut.ac.ir

Abstract

The Rosborough formulation has been mainly used so far to analyse the orbital errors of altimetry satellites. A generalization of this formulation which allows to represent any gravitational functional along the satellite orbit can be employed to process the data of the dedicated gravity field missions. The Rosborough formulation looks into the ascending and descending arcs separately. We process the first 71 days of the GOCE gradients based on the Rosborough approach. It is shown that this approach is able to produce comparable results to the so-called time-wise methodology. Moreover, it is revealed that it contains interesting characteristics which can be very advantageous in the analysis of satellite data. For instance, due to the distinction between ascending and descending data, it is naturally very sensitive to the along-orbit errors of satellite data. In the case of GOCE gradiometry data processing, the effect of the geomagnetic equator on the gradients becomes clear. As an another example, since this approach explicitly represent any functional along the orbit in the LORF using transfer coefficients, contrary to the space-wise approach, it enables one to analyse any functional, either isotropic or anisotropic, directly. In our case, a contribution analysis is employed to show the contribution of each gravitational gradient in the combined solution. In conclusion, the Rosborough approach can be considered as an viable alternative to the conventional space-wise and time-wise methodologies.

1. Rosborough formulation

The Rosborough representation is derived through transfromation of the time-wise formulation from orbital to spherical coordinates by expressing $e^{i(ku+m\Lambda)}$ in terms of ϕ,λ and I.

$$f^{\pm}(r,\phi,\lambda,I)=\sum_{n=0}^{N}\sum_{m=-n}^{n}\sum_{k=-n}^{n}\bar{K}_{nm}H^{f}_{nmk}\Psi^{\pm}_{mk}(\phi,I)e^{im\lambda}$$

$$H^{f}_{nmk}=\frac{GM}{R}\left(\frac{R}{r}\right)^{n+1}h^{f}_{nmk}\bar{F}_{nmk}(I)$$

$$\Psi^{\pm}_{mk}(\phi,I)=\frac{1}{\sin^{m+k}I\cos^{m}\phi}(\pm\sqrt{\sin^2 I-\sin^2\phi}+i\sin\phi)^k.$$

$$\left(\pm\sqrt{\sin^2 I-\sin^2\phi}-i\sin\phi\cos I\right)^m$$

The sign $\pm$ indicates distinct expressions for ascending (+) and descending tracks (−). After summation over index k we have a SH-like series:

$$f^{\pm}(r,\phi,\lambda)=\sum_{n=0}^{N}\sum_{m=-n}^{n}\bar{K}_{nm}Q^{\pm}_{nm}(\phi,I)e^{im\lambda}$$

$$Q^{\pm}_{mk}(\phi,I)=\sum_{k=-n}^{n}H^{f}_{nmk}\Psi^{\pm}_{mk}(\phi,I)$$

- Spatially mean and variable parts:

$$f^{M}=\frac{f^{+}+f^{-}}{2}\qquad f^{V}=\frac{f^{+}-f^{-}}{2}$$

$$f^{M,V}(r,\phi,\lambda)=\sum_{n=0}^{N}\sum_{m=-n}^{n}\bar{K}_{nm}Q^{M,V}_{nm}(\phi,I)e^{im\lambda}$$

2. Harmonic estimation in the Rosborough approach

Assuming the nominal orbit (constant r and I approximation) and swapping $\sum_n$ and $\sum_m$

$$f^{M,V}(\phi,\lambda)=\sum_{m=-N}^{N}C^{M,V}_{m}(\phi)e^{im\lambda}$$

with latitude lumped coefficients:

$$C^{M,V}_{m}(\phi)=\sum_{n=|m|}^{N}Q^{M,V}_{nm}(\phi,I)\bar{K}_{nm}$$

and the Rosborough base functions:

$$Q^{M,V}_{nm}(\phi,I)=\sum_{k=-n}^{n}H^{f}_{nmk}\Psi^{M,V}_{mk}(\phi,I)$$

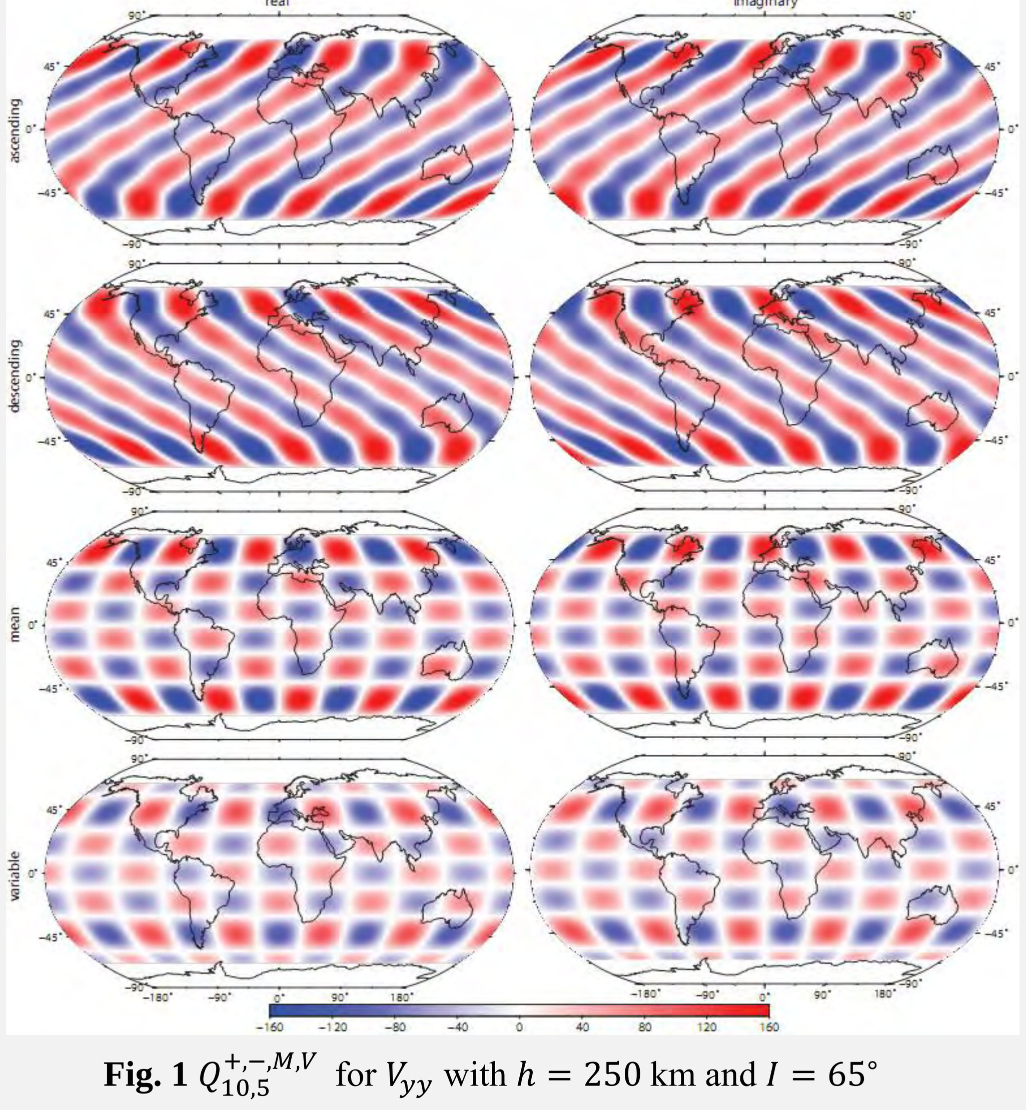

Fig. 1 $Q^{+,-,M,V}_{10,5}$ for V_{yy} with $h=250$ km and $I=65°$

3. Processing scheme for the GOCE data

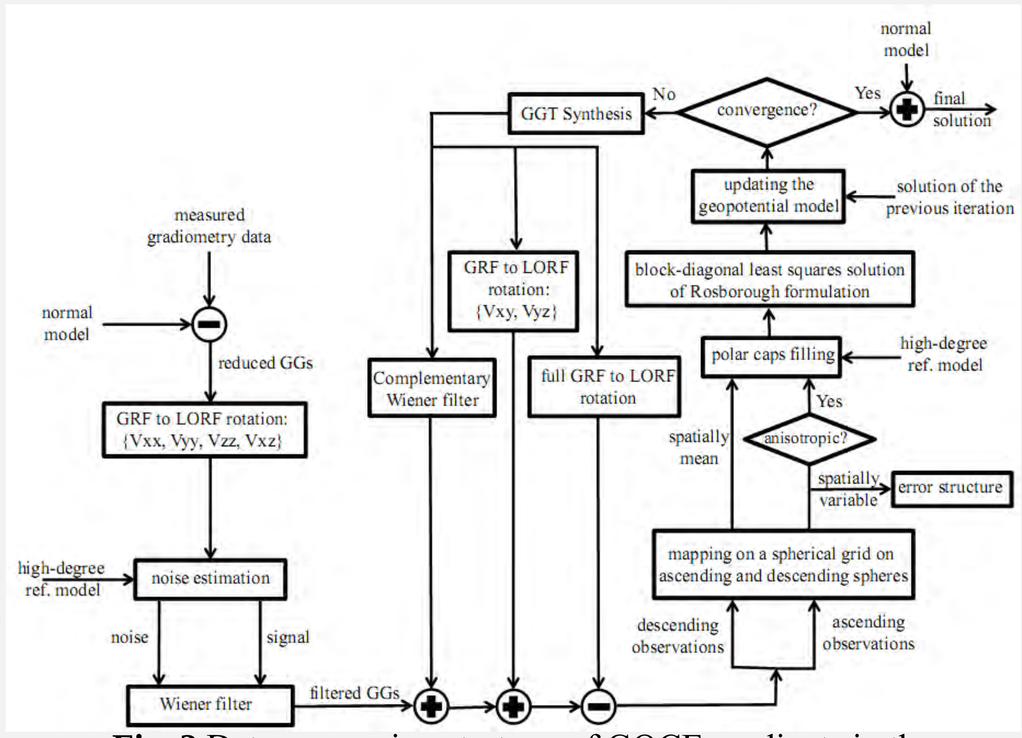

Fig. 2 Data processing strategy of GOCE gradients in the Rosborough approach

3.1. Rotation from GRF to LORF

$$\text{GRF}\rightarrow\text{IRF}\rightarrow\text{LORF}$$

3.2. Filtering the GOCE gradients: Wiener filter

$$y(t)=s(t)+n(t),\quad W(f)=\frac{S_s(f)}{S_s(f)+S_n(f)}$$

$$\hat{S}(t)=F^{-1}\{W(f).F\{y(t)\}\}$$

3.3. Gridding along-orbit data on two spheres

3.4. Iterative scheme:

3.4.1. Complementary Wiener filter

3.4.2. GRF to LORF rotation corrections

3.4.3. Gridding error mitigation

4. Real GOCE processing

- The first 71 days (01.11.2009–10.01.2010) of reprocessed EGG_NOM_2 data (V_{xx},V_{yy},V_{zz}).
- EGM08 to d/o 250 is used in Wiener filtering, polar gaps filling, etc.
- *A priori* model is EGM08 to d/o 80.

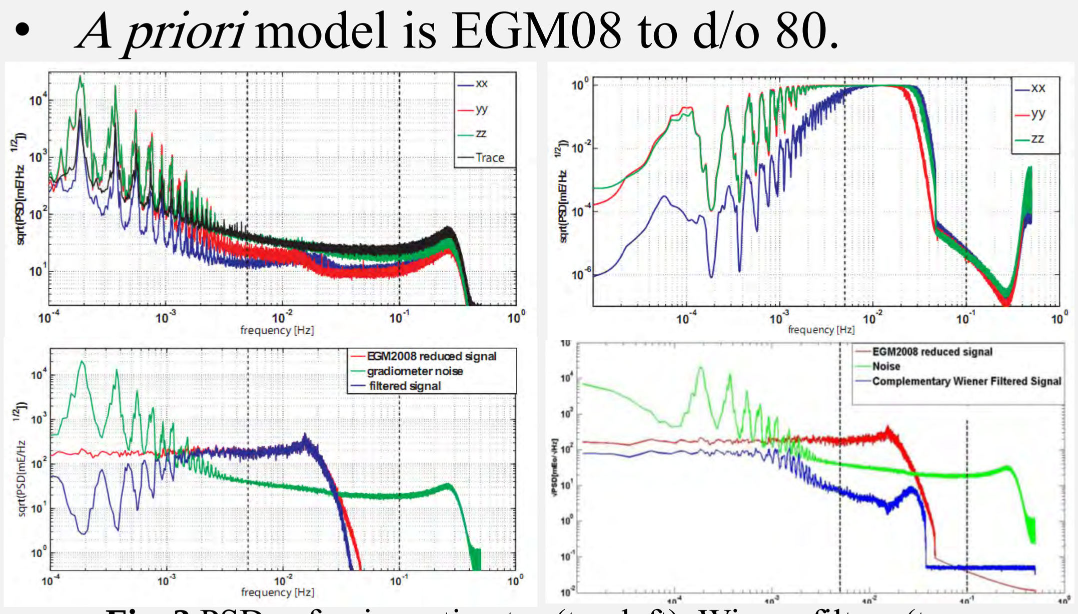

Fig. 3 PSDs of noise estimates (top left), Wiener filters (top right), Wiener filtered T_{zz} (bottom left) and complementary Wiener filtered T_{zz} (bottom right)

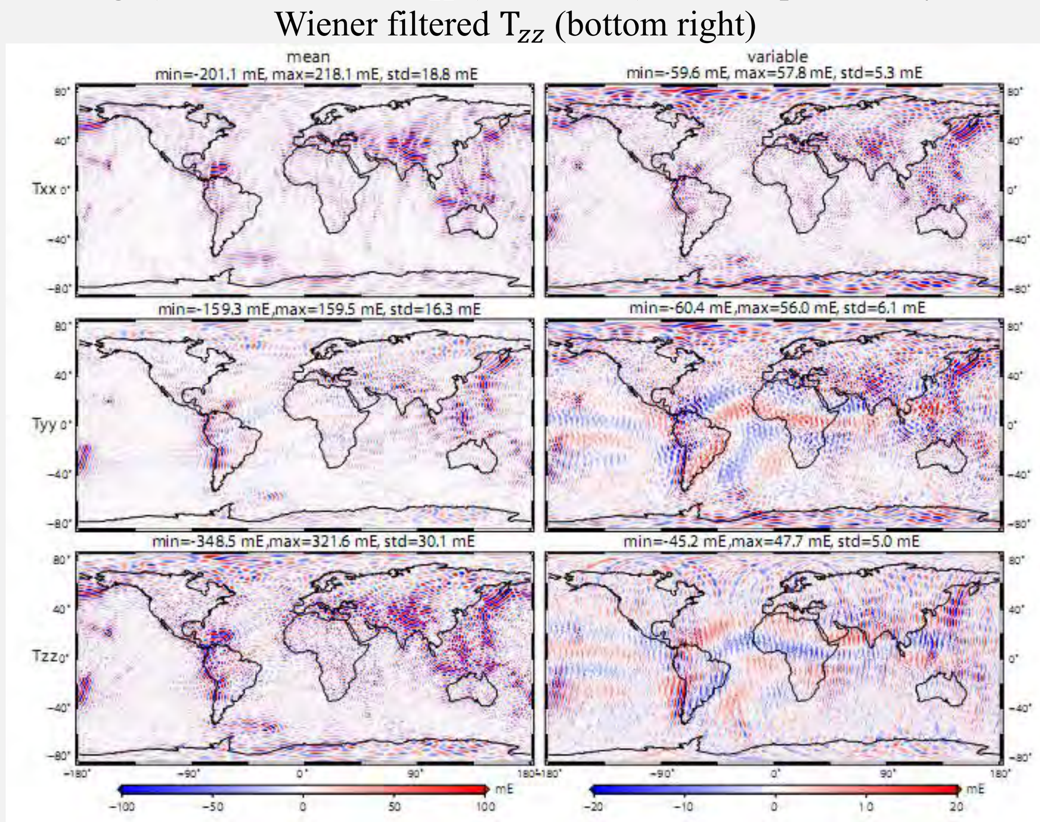

Fig. 4 Spatially mean and variable parts of gradients. Note the geomagnetic equator effect in the variabe parts T_{yy} and T_{zz}

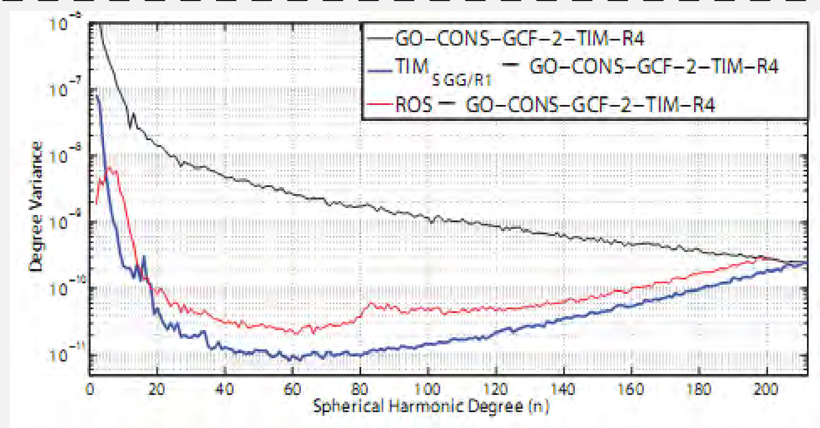

Fig. 5 Error degree variance of Rosborough and time-wise solutions

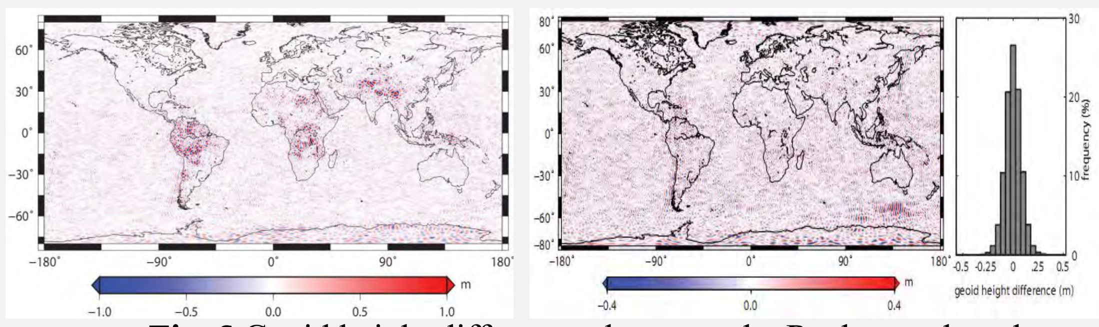

Fig. 6 Geoid height differences between the Rosborough and EGM08 (left) and time-wise models (right) from degree 80 to 180

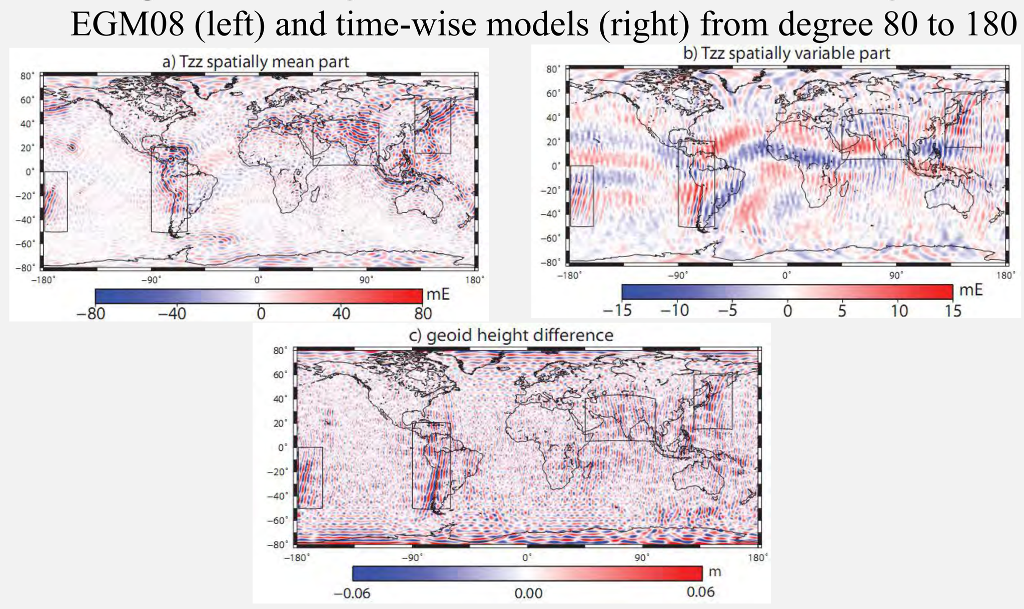

Fig. 7 Filtered T_{zz} spatially mean (top left) and variable (top right) parts and geoid height differences (bottom) using a 500 km Gaussian filter

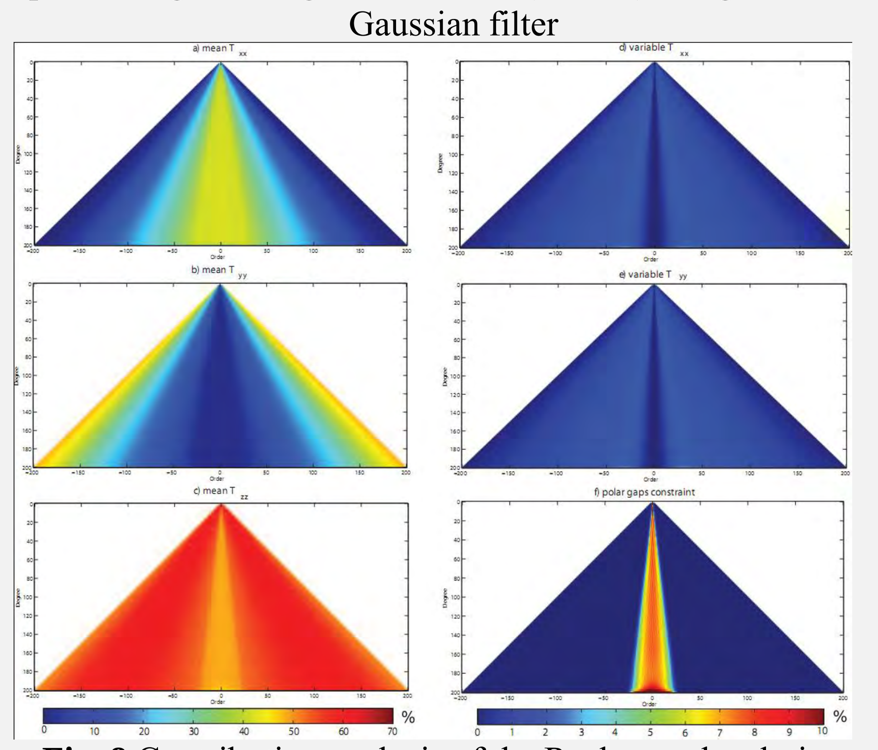

Fig. 8 Contribution analysis of the Rosborough solution

Ghobadi-Far K, Sharifi MA, Sneeuw, N (2015) GOCE gradiometry data processing using the Rosborough approach; Journal of Godesy; DOI: 10.1007/s00190-015-0849-6

5. Conclusions

- ✓ Solution up to N_{max}=200 from 71 days of GOCE gradients with performance of 8 cm geoid RMS difference w.r.t the time-wise model in the relevant bandwidth
- ✓ Relative contributions of T_{xx}, T_{yy} and T_{zz} are about 20%, 20% and 57%.
- ✓ Error analysis of satellite data based on spatially variable parts: geomagnetic equator effect, track-specific errors, tidal mismodeling.
- ✓ Lower accuracy of the Rosborough solution mostly due to gridding error
- ✓ The Rosborough method is a ***complementary*** method to the conventional approaches