Storage and discharge estimation of Danube basin by least-squares prediction

Universität Stuttgart

J. Zhang, M. J. Tourian, N. Sneeuw

Institute of Geodesy, University of Stuttgart

zhang@gis.uni-stuttgart.de

GIS

Introduction

The total storage changes can be observed by GRACE. However, GRACE has already outlived its predicted lifetime by many years, thus increasing the risk of a possible gap in regular observations of the total water storage changes until GRACE Follow-On becomes operational in 2017. The variation in water storage is the result of interaction between hydrological fluxes e.g. river discharge. The river discharge has been monitored for long time. The relationship between river discharge and storage can be characterized as a Linear Time Invariant (LTI) System (Riegger and Tourian, 2014). Therefe, using statistical methods to model the relationship of storage and discharge might help to bridge such possible gap. In this research, with observations of river discharge, we explore the viability of least-squares prediction at catchment scale. The storage change of Danube basin is predicted by river discharge and compared with the observations from GRACE. Furthermore, to test the capability of assimilation with storage and discharge, the river discharge of Danube basin is predicted by both discharge from different catchments and storage observed by GRACE, as well. The results are validated against in-situ measurements and compared with prediction only by storage change.

Methodology

Least-Squares Prediction

According to Moritz (1980), the least-squares prediction makes use of the covariance information of signals to predict a signal for target out of observations statistically, which can be represented by

$$\hat{s} = H \cdot l\,, \quad e = s - \hat{s}\,, \tag{1}$$

where H is prediction matrix which needs to be determined, l and $\hat{s}$ are observations and predictions, respectively. s is true signal and e is prediction errors between true and predicted signals.
To minimize the error e in order to obtain optimal estimation, the H can be computed as

$$H = C_{sl} \cdot C_{ll}^{-1}\,, \tag{2}$$

where C_{sl} is the cross-covariance matrix of signal s and l, C_{ll} is the covariance matrix of signal l.

There is always a problem that s is unknown and therefore we cannot obtain C_{sl} directly. To determine the cross-covariance matrix C_{sl}, we set up a training period, for which we use legacy data or models. Due to the irregular observations, in this case, we compute the covariance matrix pairwise for each two time series.

In this study, to avoid rank deficiency of covariance matrix, we remove the monthly mean (mean of Jan., Feb., etc.) from observations and only use the residuals in both training period and prediction process. We have used GRACE data from German Geoscience Research Center (GFZ). $C_{2,0}$ is replaced by coefficient from SLR. The GRACE data are then filtered by a Gaussian smoothing filter with 350 km radius and a destriping filter. River discharge data are provided by Global Runoff Data Center (GRDC).

Results

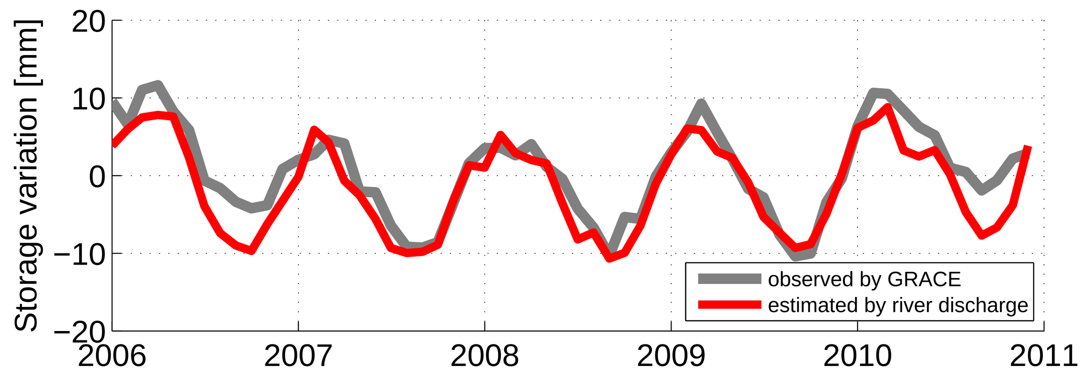

Figure 1: Storage variation estimation by river discharge

In Figure 1, the storage is predicted by river discharge of Danube basin (in red color shown in Figure 2). Compared with observations from GRACE, it shows quite good consistency in phase and amplitude. The results reveal a linear relationship between storage and discharge.

To test the viability of river discharge to fill the gaps of mass variation, Amazon, Danube and Mississippi basins have been analyzed. We used the discharge measurements of targeted catchment to predict the storage change. The Nash-Sutcliffe Efficiency (NSE) and NSE with respect to monthly mean ($\widetilde{\text{NSE}}$) are calculated for evaluation. Root Mean Square Error (RMSE) of the prediction is computed as well. Performances of prediction in these catchments behave differently.

Table 1: Performance of storage prediction of different catchments

Catchments	Observations	NSE	$\widetilde{\text{NSE}}$	RMSE (mm)
Amazon	discharge	0.84	-1.79	22.8
Danube	discharge	0.73	0.04	3.0
Mississippi	discharge	0.78	-0.47	6.4

Results

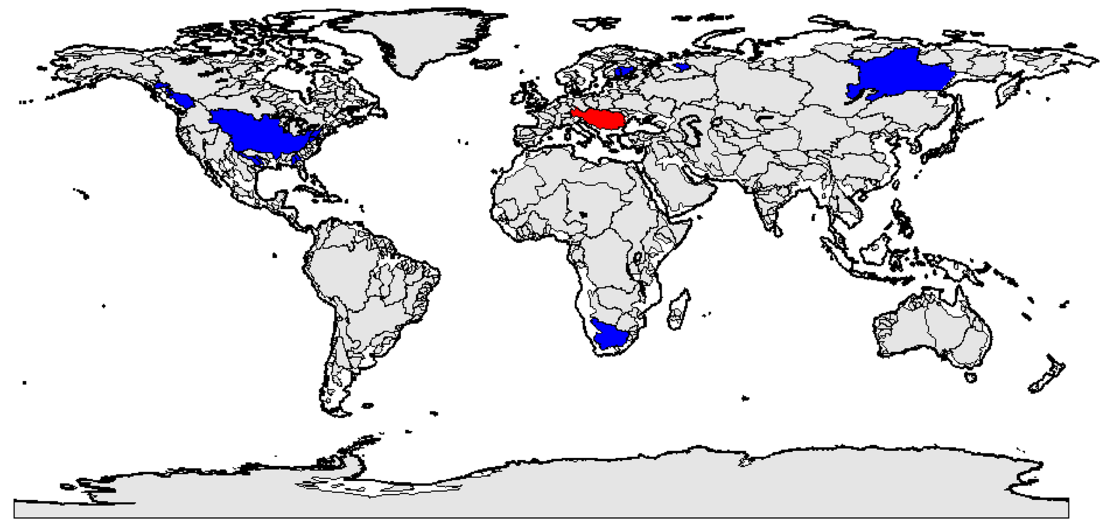

Figure 2: Distribution of catchments used in prediction

In this study, we choose a 36-month training period (January 2003 to December 2005) for GRACE and discharge observations. We predict the river discharge and storage change from 2006 to 2010. The relationship between storage and discharge is shown in Figure 3.

Table 2: Performance of discharge prediction of Danube basin

Observations	NSE	$\widetilde{\text{NSE}}$	RMSE (mm/month)
Discharge	0.17	-0.29	8.5
Storage	0.64	0.44	5.6
Discharge and Storage	0.58	0.35	6.0

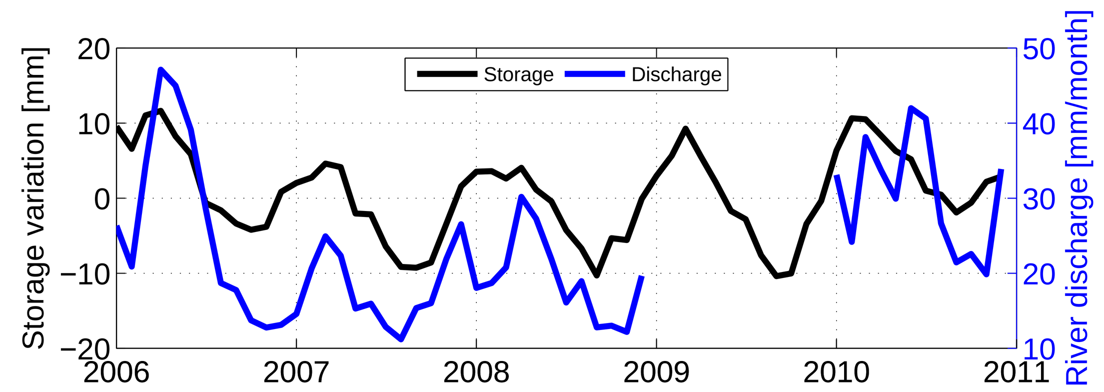

Figure 3: River discharge from GRDC and storage from GRACE of Danube basin

In Figure 4, the river discharge of Danube basin (in red color shown in Figure 2) is predicted epoch-wise by discharge from 10 catchments, and also predicted by storage in Danube from GRACE. The prediction from discharge is worse than monthly mean but prediction from storage shows quite good performance (as shown in Table 2). After assimilating discharge and storage together, it improves the quality of discharge prediction, and can capture the high and low amplitudes, which complements the prediction from storage.

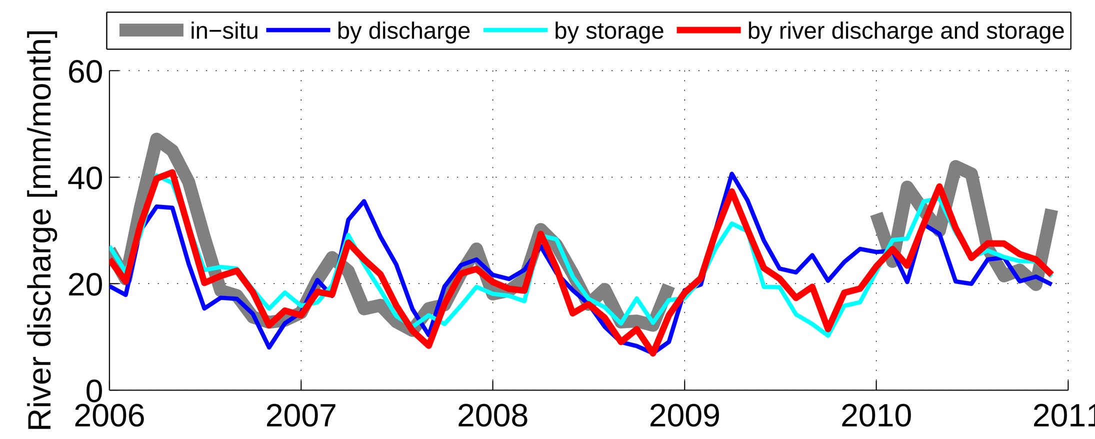

Figure 4: River discharge estimation of Danube basin by storage and river discharge assimilation

Conclusion and Discussion

- Storage variation and river discharge show high dependency, and river discharge can be used to bridge the possible gaps at catchment scale.
- Different performance of storage prediction in terms of $\widetilde{\text{NSE}}$ for the studied catchments might due to the strong seasonality and cyclo-stationarity of signals.
- Prediction of the storage change for sub-catchment and even smaller scales is still a remaining problem.
- Our analysis revealed that the prediction quality depends on the number and selection of catchments. Therefore the climate classes could also be considered for our future research.

References

[1] H. Moritz. *Advanced Physical Geodesy*. Wichmann, 1980.

[2] J. Riegger and M. J. Tourian. Characterization of runoff-storage relationships by satellite gravimetry and remote sensing. *Water Resources Research*, 50, 2014.