Cubature Particle filter applied in a tightly-coupled GPS/INS navigation system

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Main Contents

The Cubature Particle Filter

➢The Gaussian Particle Filter

- Use the Gaussian distribution to approximate the importance density function in the Particle Filter
- No particles' degeneration
- No need in resampling the particles
- Reduce the computational burden

➢The Cubature Kalman Filter

- The heart is the spherical-radial cubature rules
- Use 2n equal weighted cubature points
- A third-order approximation to the nonlinear system

➢The Cubature Particle Filter

- Use the state estimated by the CKF to form the importance density function

The Cubature Kalman Filter

- Time update

$$
\begin{cases}
P_{k-1|k-1} = S_{k-1|k-1} S_{k-1|k-1}^{T} \\
\chi_{k-1|k-1} = S_{k-1|k-1}\xi + x_{k-1|k-1} \\
\chi_{k-1|k-1}^{*} = f(\chi_{k-1|k-1}, u_{k-1}) \\
x_{k|k-1} = \frac{1}{m}\sum_{i=1}^{m} \chi_{i,k-1|k-1}^{*} \\
P_{k|k-1} = \frac{1}{m}\sum_{i=1}^{m} \chi_{i,k|k-1}^{*}\chi_{i,k|k-1}^{*T} - x_{k|k-1}x_{k|k-1}^{T} + Q_{k-1}
\end{cases}
$$

- Measurement update

$$
\begin{cases}
P_{k|k-1} = S_{k|k-1} S_{k|k-1}^{T} \\
\chi_{k|k-1} = S_{k|k-1}\xi + x_{k|k-1} \\
Z_{k-1|k-1} = h(\chi_{k-1|k-1}, u_{k-1}) \\
z_{k|k-1} = \frac{1}{m}\sum_{i=1}^{m} Z_{i,k-1|k-1} \\
P_{zz,k|k-1} = \frac{1}{m}\sum_{i=1}^{m} Z_{i,k|k-1}Z_{i,k|k-1}^{T} - z_{k|k-1}z_{k|k-1}^{T} + R_{k} \\
P_{xz,k|k-1} = \frac{1}{m}\sum_{i=1}^{m} \chi_{i,k|k-1}Z_{i,k|k-1}^{T} - x_{k|k-1}z_{k|k-1}^{T} \\
K_{k} = P_{xz,k|k-1}P_{zz,k|k-1}^{-1} \\
x_{k|k} = x_{k|k-1} + K_{k}(z_{k} - z_{k|k-1}) \\
P_{k|k} = P_{k|k-1} - K_{k}P_{zz,k|k-1}K_{k}^{T}
\end{cases}
$$

The Cubature Particle Filter

- The particles will be generated using the CKF a posteriori estimates as

$$X_{k,i} \sim N(x_{k|k}, P_{k|k})$$

- The weights can be calculated as

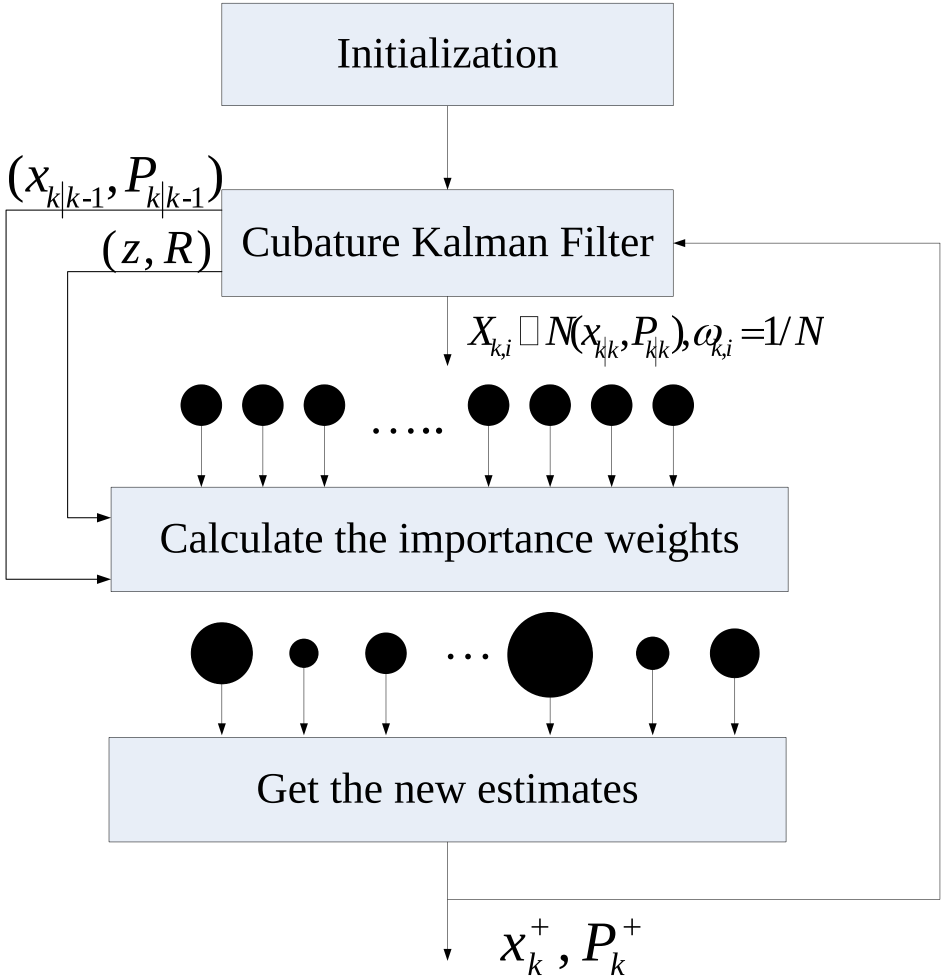
$$\omega(X_{k,i}) = \frac{p(z_k | X_{k,i}) N(X_{k,i} | x_{k|k-1}, P_{k|k-1})}{N(X_{k,i} | x_{k|k}, P_{k|k})} \qquad \tilde{\omega}(X_{k,i}) = \frac{\omega(X_{k,i})}{\sum_{i=1}^{N} \omega(X_{k,i})}$$

- The states and covariance can be estimated as

$$x_k^+ = \sum_{i=1}^{N} \tilde{\omega}(X_{k,i}) X_{k,i}$$

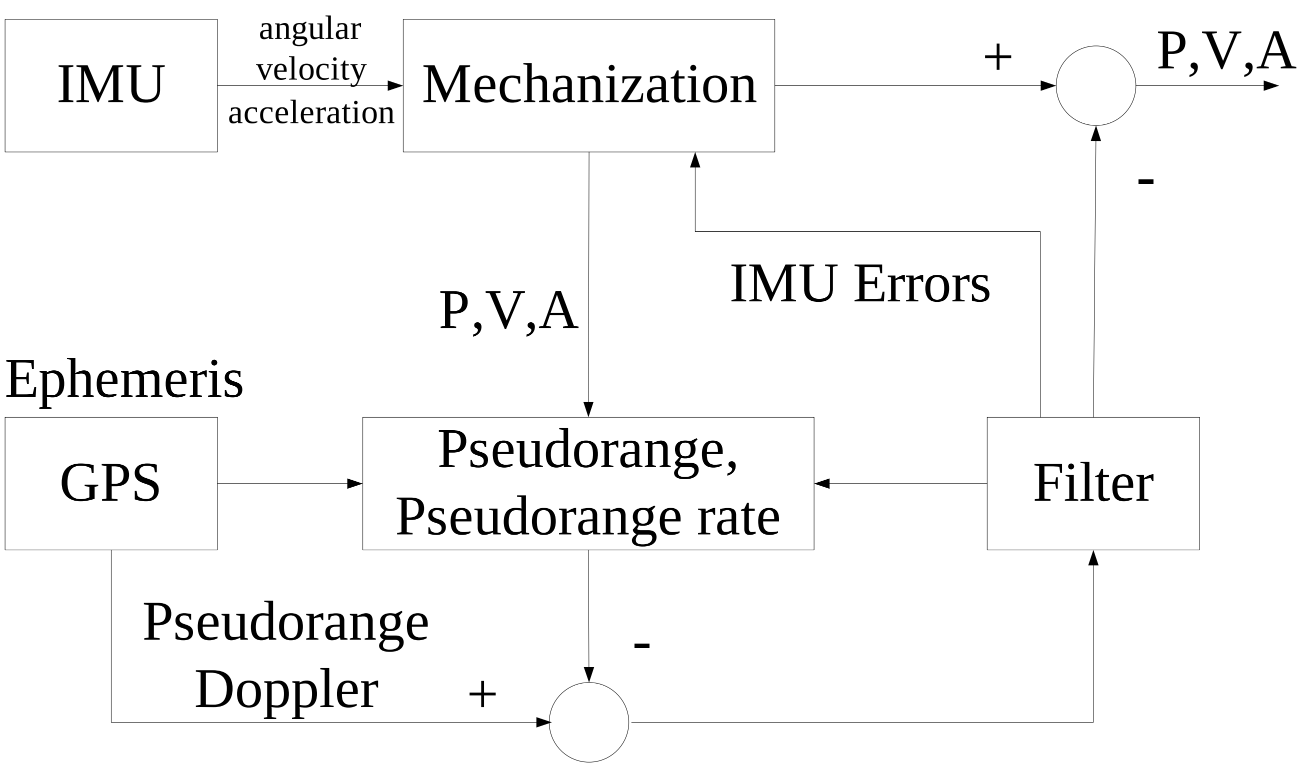
$$P_k^+ = \sum_{i=1}^{N} \tilde{\omega}(X_{k,i}) [X_{k,i} - x_k^+][X_{k,i} - x_k^+]^T$$

The Cubature Particle Filter



GPS/INS tightly-coupled navigation system

- System structure



GPS/INS tightly-coupled navigation system

- State vector

$$x = [\delta\alpha, \delta\beta, \delta\gamma, \delta v_x, \delta v_y, \delta v_z, \delta l, \delta b, \delta h, \nabla_x, \nabla_y, \nabla_z, \varepsilon_x, \varepsilon_y, \varepsilon_z, c\delta t, c\dot{\delta} t]$$

  - The difference between the true value and estimated value
  - $\delta\alpha, \delta\beta, \delta\gamma$:the attitude error
  - $\delta v_x, \delta v_y, \delta v_z$:the velocity error
  - $\delta l, \delta b, \delta h$:the position error (longitude, latitude and height)
  - $\nabla_x, \nabla_y, \nabla_z$:the gyro drift
  - $\varepsilon_x, \varepsilon_y, \varepsilon_z$:the acceleration bias
  - $c\delta t$:the receiver clock offset expressed in meters
  - $c\dot{\delta} t$: the receiver clock drift expressed in meters

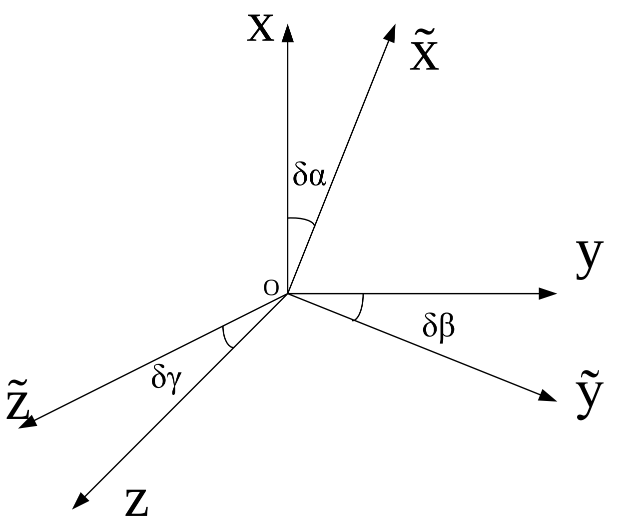
GPS/INS tightly-coupled navigation system

- Transition function

$$\begin{cases} \dot{\phi} = (I - C_n^{\tilde{n}})\tilde{\omega}_{in}^n + C_n^{\tilde{n}}\delta\omega_{in}^n - C_b^{\tilde{n}}\delta\omega_{ib}^b \\ \delta\dot{v}^n = [I - C_n^{\tilde{n}}]\tilde{C}_b^{\tilde{n}}\tilde{f}^b + C_b^{\tilde{n}}\delta f^b - (2\delta\omega_{ie}^n + \delta\omega_{en}^n) \times v^n - (2\tilde{\omega}_{ie}^n + \tilde{\omega}_{en}^n) \times \delta v^n + \delta g^n \\ \delta\dot{L} = \dfrac{\delta v_N^n}{M+h} - \dfrac{\delta h v_N^n}{(M+h)^2} \\ \delta\dot{b} = \dfrac{\delta v_E^n \sec L}{N+h} + \dfrac{\delta L v_E^n \tan L \sec L}{N+h} - \dfrac{\delta h v_E^n \sec L}{(N+h)^2} \\ \delta\dot{h} = \delta v_U^n \\ \dot{\nabla} = -\beta\nabla \\ \dot{\varepsilon} = -\beta\varepsilon \end{cases}$$

GPS/INS tightly-coupled navigation system

- An approximation to the attitude error
  - The attitude error is treated as the rotational angle between the true and estimated navigation frames
- The direction cosine matrix can be used
- Different from the psi-angle expression



$$C_n^{\tilde{n}} = \begin{bmatrix} \cos\delta\beta\cos\delta\gamma - \sin\delta\beta\sin\delta\alpha\sin\delta\gamma & \cos\delta\beta\sin\delta\gamma + \sin\delta\beta\sin\delta\alpha\cos\delta\gamma & -\sin\delta\beta\cos\delta\alpha \\ -\cos\delta\alpha\sin\delta\gamma & \cos\delta\alpha\cos\delta\gamma & \sin\delta\alpha \\ \sin\delta\beta\cos\delta\gamma + \cos\delta\beta\sin\delta\alpha\sin\delta\gamma & \sin\delta\beta\sin\delta\gamma - \cos\delta\beta\sin\delta\alpha\cos\delta\gamma & \cos\delta\beta\cos\delta\alpha \end{bmatrix}$$

GPS/INS tightly-coupled navigation system

- Measurement update
  - Pseudo-range and Pseudo-range rate
  - Difference between the observation and the estimated value
  - Position is transferred from ECEF to LBH

Pseudo-range
$$r_j = \sqrt{(x_{s,j} - x)^2 + (y_{s,j} - y)^2 + (z_{s,j} - z)^2}$$

Difference
$$\delta\rho = c\delta t_r - e_{j1}\delta x - e_{j2}\delta y - e_{j3}\delta z$$

Pseudo-range rate
$$\dot{r}_j = e_{j1}(\dot{x}_{s,j} - \dot{x}) + e_{j1}(\dot{y}_{s,j} - \dot{y}) + e_{j1}(\dot{z}_{s,j} - \dot{z})$$

Difference
$$\delta\dot{\rho}_j = g_{j1}\delta x + g_{j2}\delta y + g_{j3}\delta z + e_{j1}\delta\dot{x} + e_{j2}\delta\dot{y} + e_{j3}\delta\dot{z} + c\delta\dot{t}_r$$

Experiment

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- MEMS IMU

	Gyroscope	Acceleration
Bias	0.1 ° /s	0.1 mg
Noise	2 $^\circ/\sqrt{h}$	36 $\mu g/\sqrt{Hz}$

- RLG IMU

	Gyroscope	Acceleration
Bias	0.003 ° /h	25 µg
Noise	0.002 $^\circ/\sqrt{h}$	8 $\mu g/\sqrt{Hz}$

Experiment


Google
Image © 2013 AeroWest
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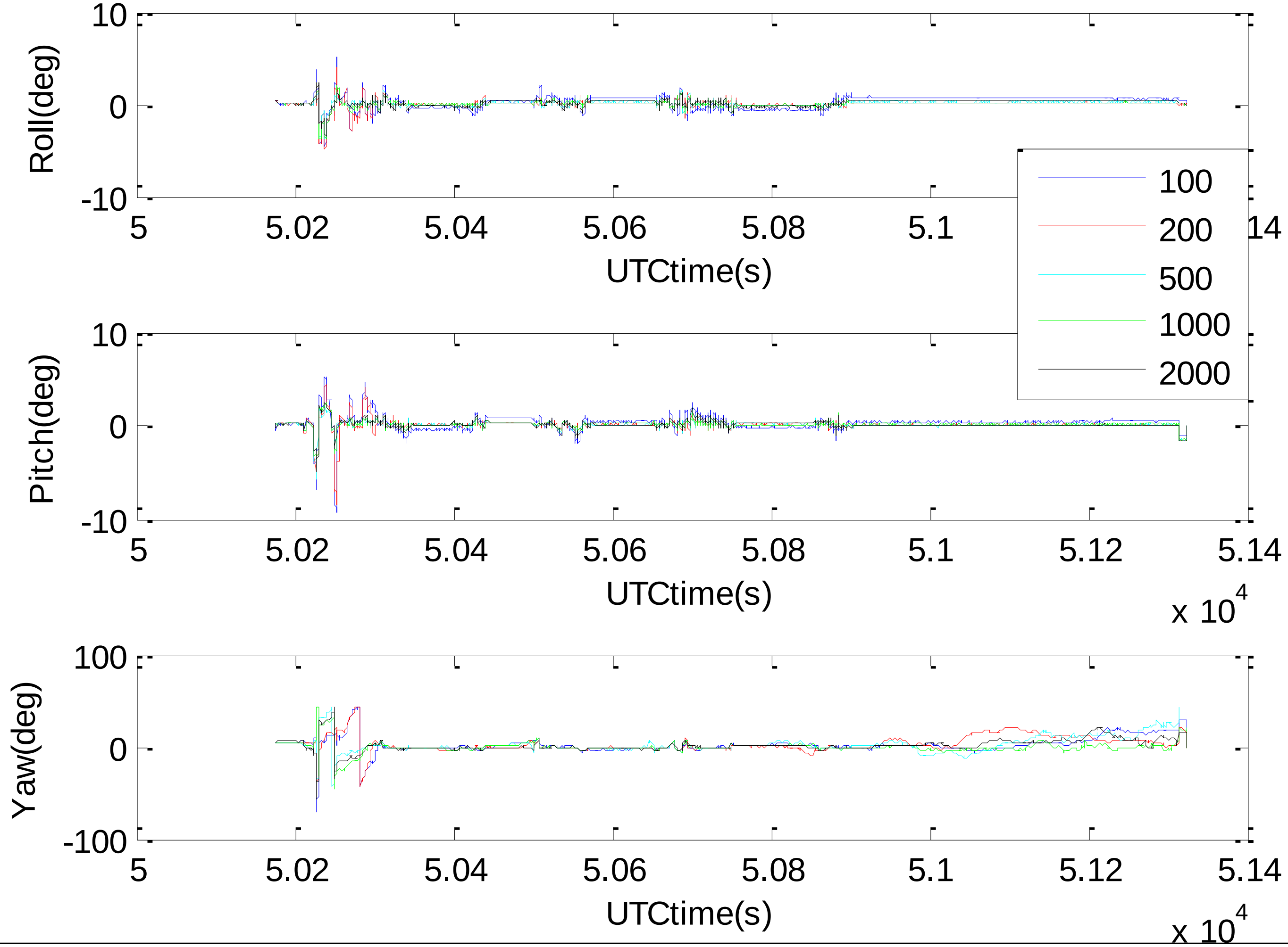
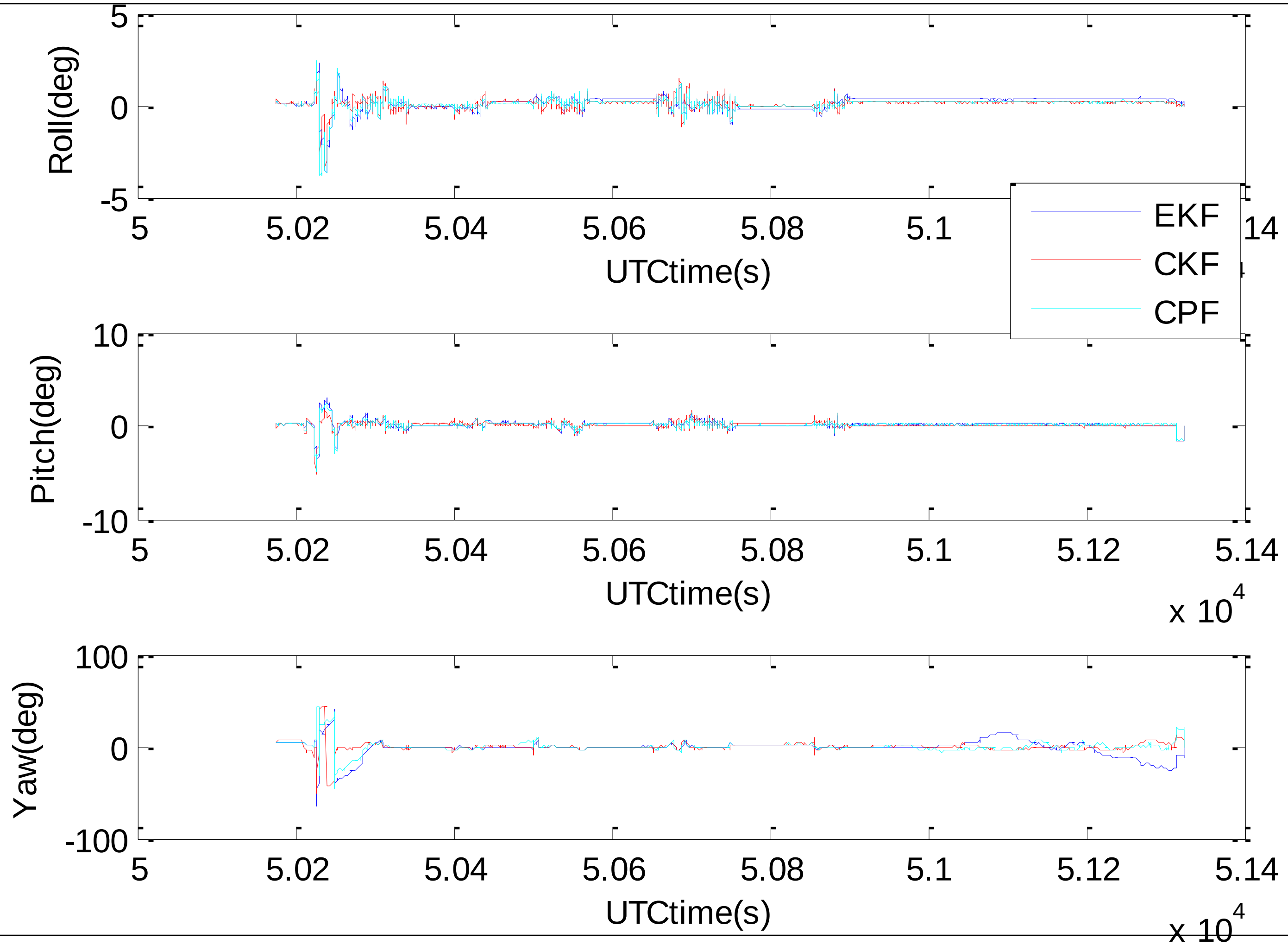
Comparison among different particle's number--Attitude



Comparison among different particle's number--Attitude

	Roll(deg)			Pitch(deg)			Yaw(deg)		
	Max	Mean	RMS	Max	Mean	RMS	Max	Mean	RMS
100	5.0113	0.6303	0.7166	9.3864	0.5758	0.8762	69.460	5.0155	8.8590
200	4.8550	0.2589	0.4959	8.5296	0.2792	0.7031	55.355	5.6760	8.7922
500	3.6620	0.2578	0.3785	5.8755	0.2281	0.4332	63.874	5.7344	8.8378
1000	3.8888	0.2119	0.3798	4.9648	0.2392	0.4191	52.045	3.2600	6.3755
2000	3.3746	0.3456	0.4069	4.8089	0.2358	0.4259	57.756	4.0746	6.4198

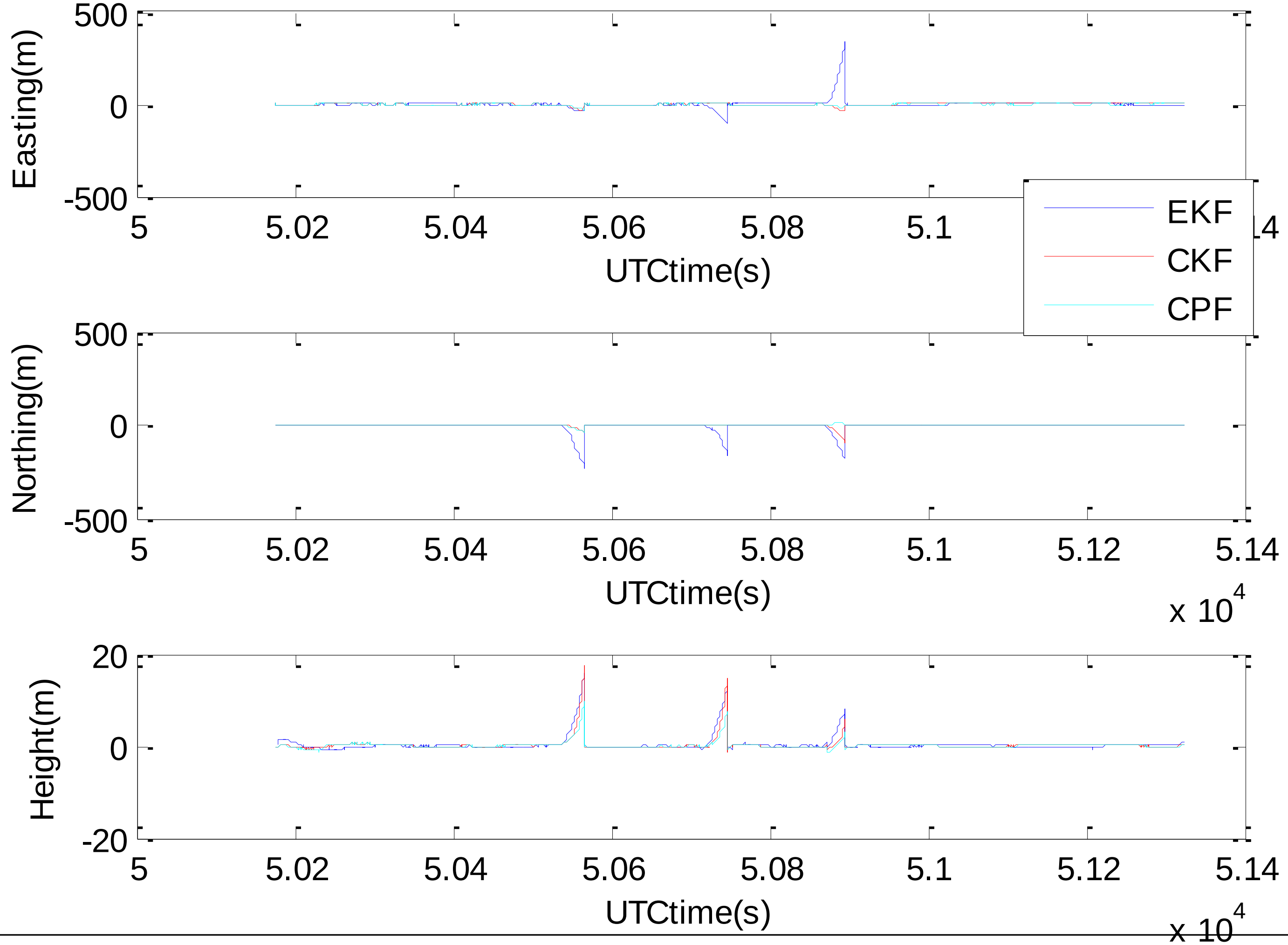
Comparison among different filtering methods--Attitude



Comparison among different filtering methods--Attitude

	Roll(deg)			Pitch(deg)			Yaw(deg)		
	Max	Mean	RMS	Max	Mean	RMS	Max	Mean	RMS
EKF	3.7100	0.3054	0.3987	4.8508	0.2598	0.4511	64.525	4.9888	9.0527
CKF	3.4186	0.2214	0.3890	5.2898	0.2088	0.4047	52.511	3.0077	6.4198
CPF	3.8888	0.2119	0.3798	4.9648	0.2392	0.4191	52.045	3.2600	6.3755

Coasting Performance



Coasting-Maximum Position difference

	Easting(m)			Northing(m)			Height(m)		
	CT 1	CT 2	CT 3	CT 1	CT 2	CT 3	CT 1	CT 2	CT 3
EKF	34.434	105.63	335.90	231.16	164.50	174.55	15.569	11.997	8.1558
CKF	39.984	1.3947	35.570	34.938	4.4921	90.350	17.732	14.694	5.8107
CPF	30.781	5.8149	21.683	36.967	1.6557	16.514	9.5098	7.6455	3.2268

Conclusion & Future work

- **Conclusion**
  - The CPF performs better than the EKF
  - The CPF shows similar performance with the CKF using Gaussian distribution
  - The CKF can ease the curse of dimensionality, but can’t eliminate it
  - High computation burden
- **Future work**
  - Rao-Blackwellized Cubature Particle Filter
  - Gaussian-sum Cubature Particle Filter

Thank you very much for the attention!!

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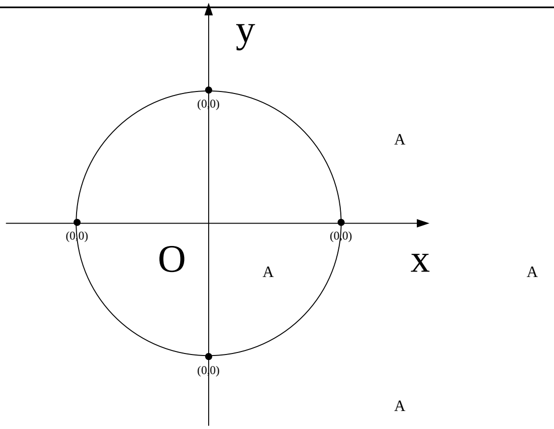
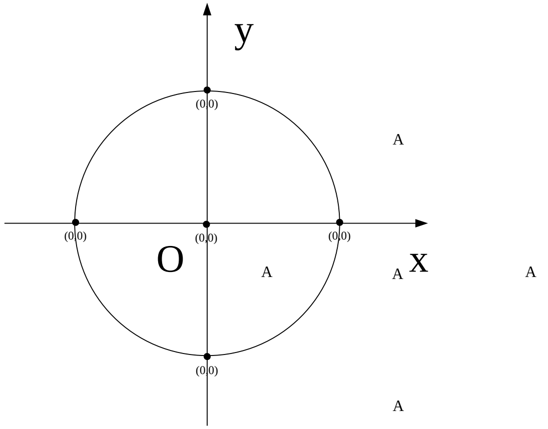
PSGD

Comparison between the UKF and the CKF

- Difference
  - ➢ CKF: 2n Cubature points
  - UKF: 2n+1 Sigma points
  - ➢ CKF: all the weights are positive, no possibility in negative definite
  - UKF: has the possibility in negative definite
  - ➢ CKF: more suitable for higher-order system
  - UKF: more suitable for lower-order system
  - ➢ CKF: proved in theory
  - UKF: based on the assumption
  - ➢ CKF: only has one parameter to tune
  - UKF: has three parameters to tune
- Similarity
  - A third–order approximation to the nonlinear system
  - The CKF can be treated as a special case of the UKF
  - Gaussian filter

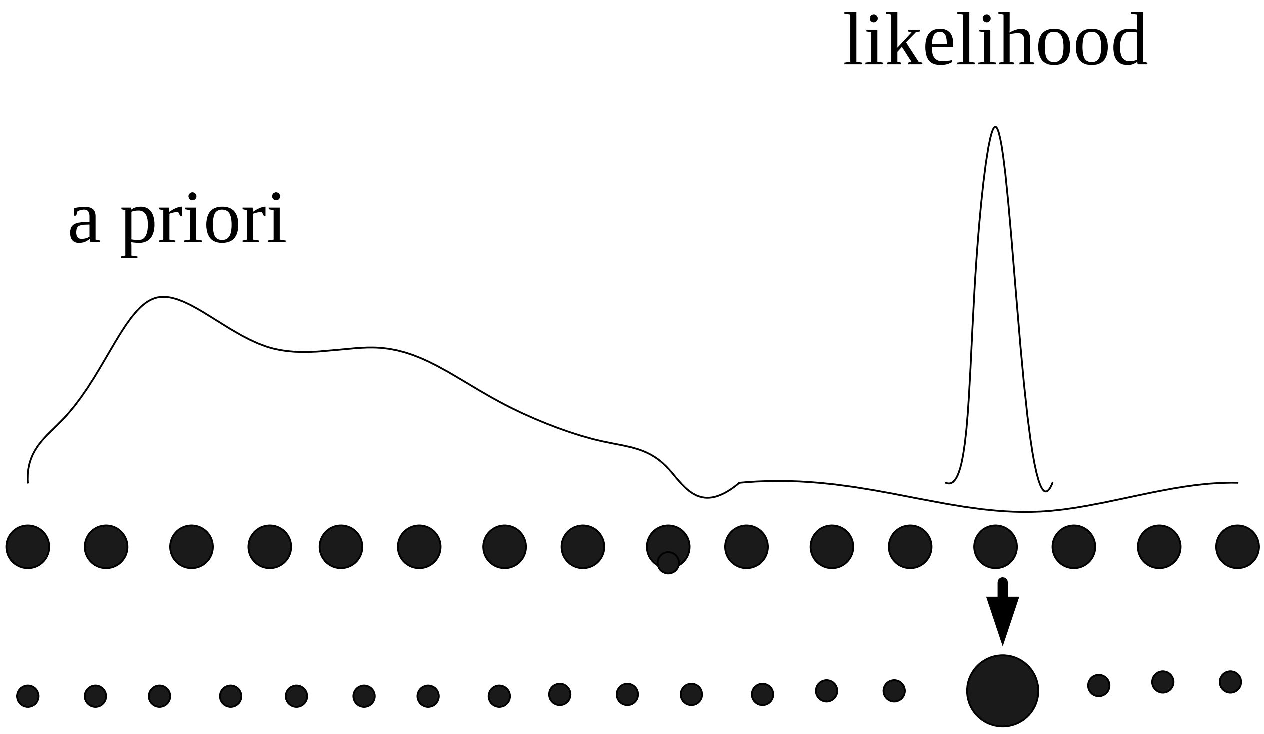


CKF Cubature points distribution



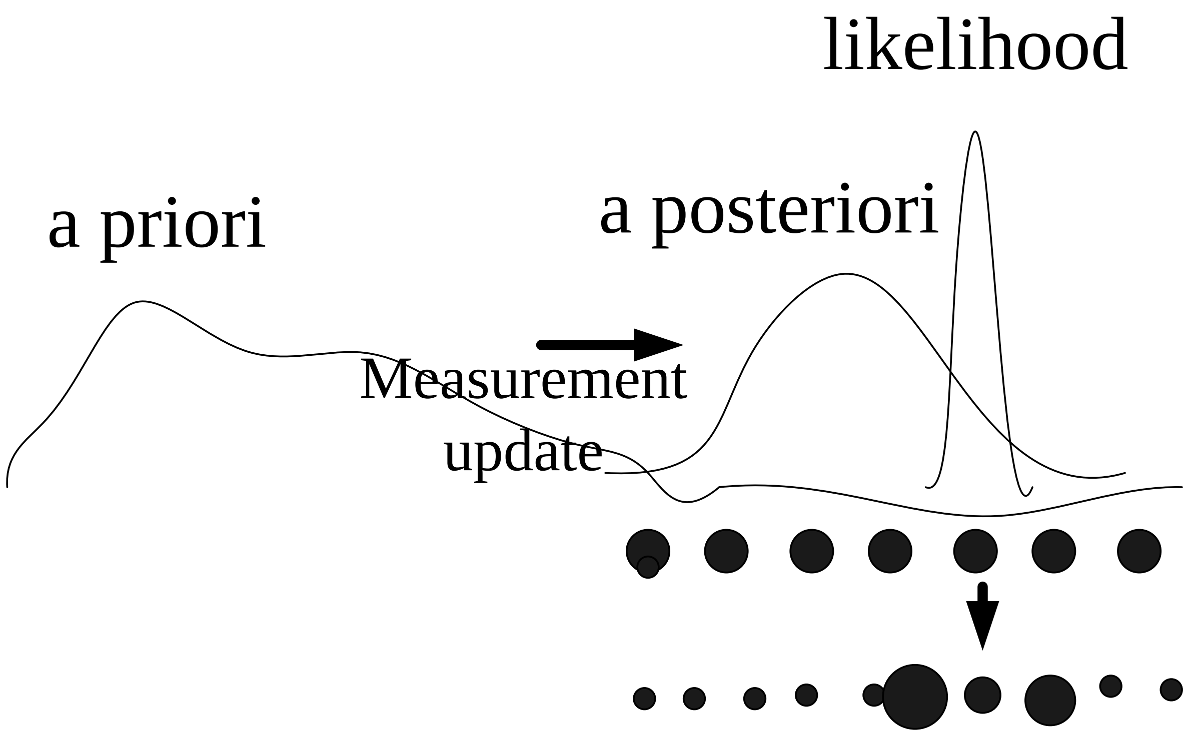
UKF Sigma points distribution

The function of the CKF in the CPF



Bootstrap Particle Filter

The function of the CKF in the CPF



Cubature Particle Filter

The reason of the jumps in the beginning stage in the attitude

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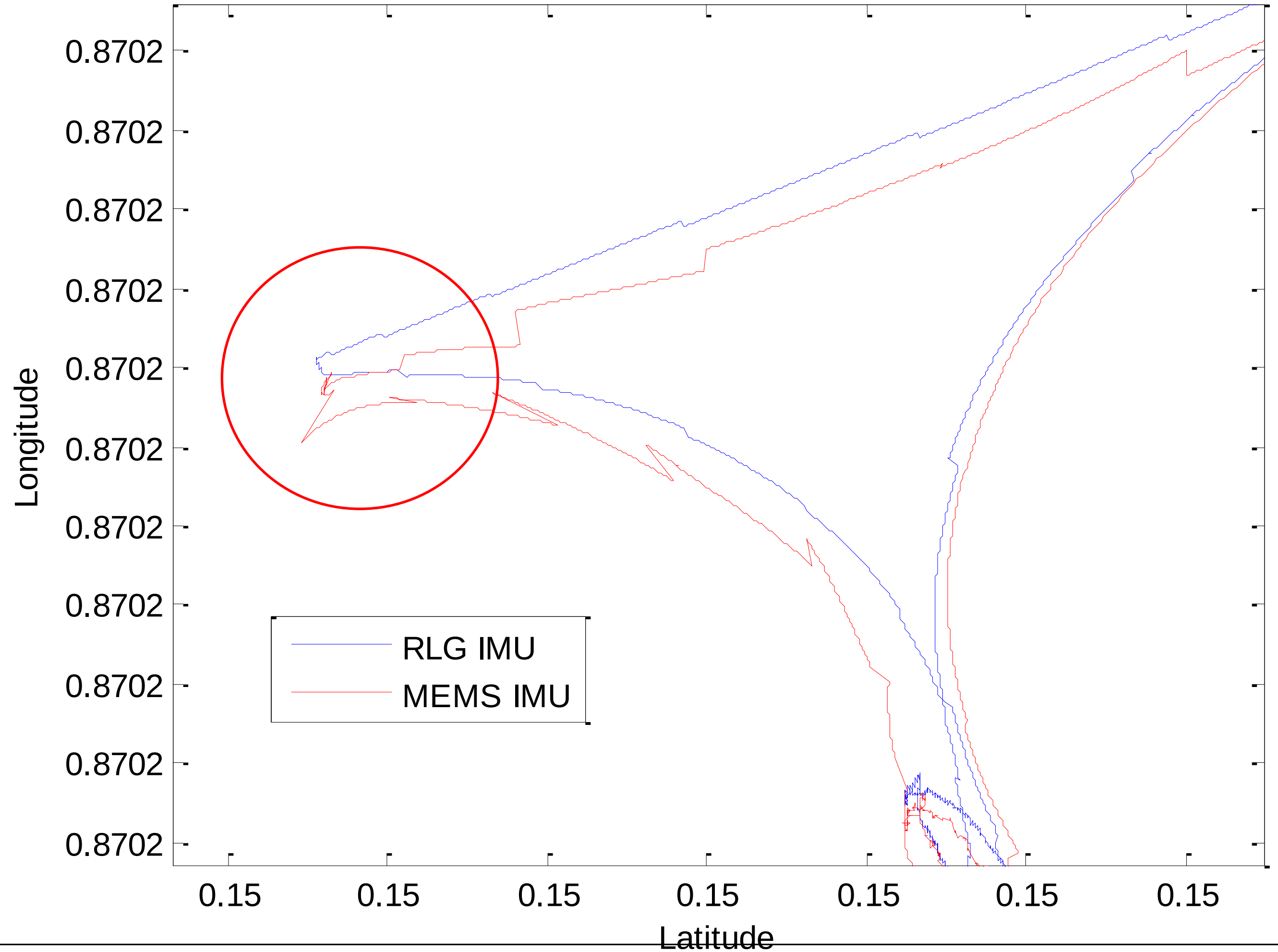

PSGD

The observability of the attitude error

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	$a_E=0$	$a_E\neq0$	S maneuvering	Turning
$\delta\alpha$	1.2e-7	1.2e-7	1.7e-7	1.3e-7
$\delta\beta$	1.2e-7	1.2e-7	1.7e-7	1.3e-7
$\delta\gamma$	1.4e-3	9.3e-4	8.5e-4	2.8e-5

PSGD