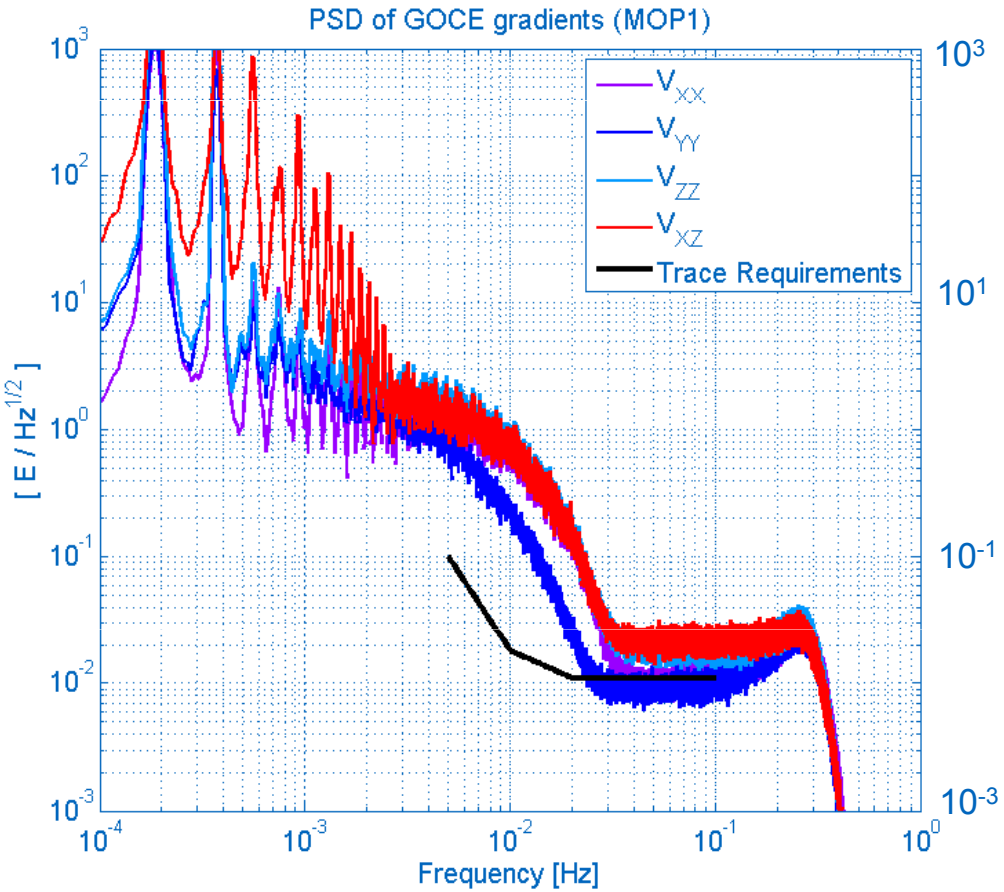
Gravity from integrated GOCE gradients

Michael Murböck,
Roland Pail, Reiner Rummel

Motivation

- V_{xz} one of four good components
- Higher noise in lower frequencies

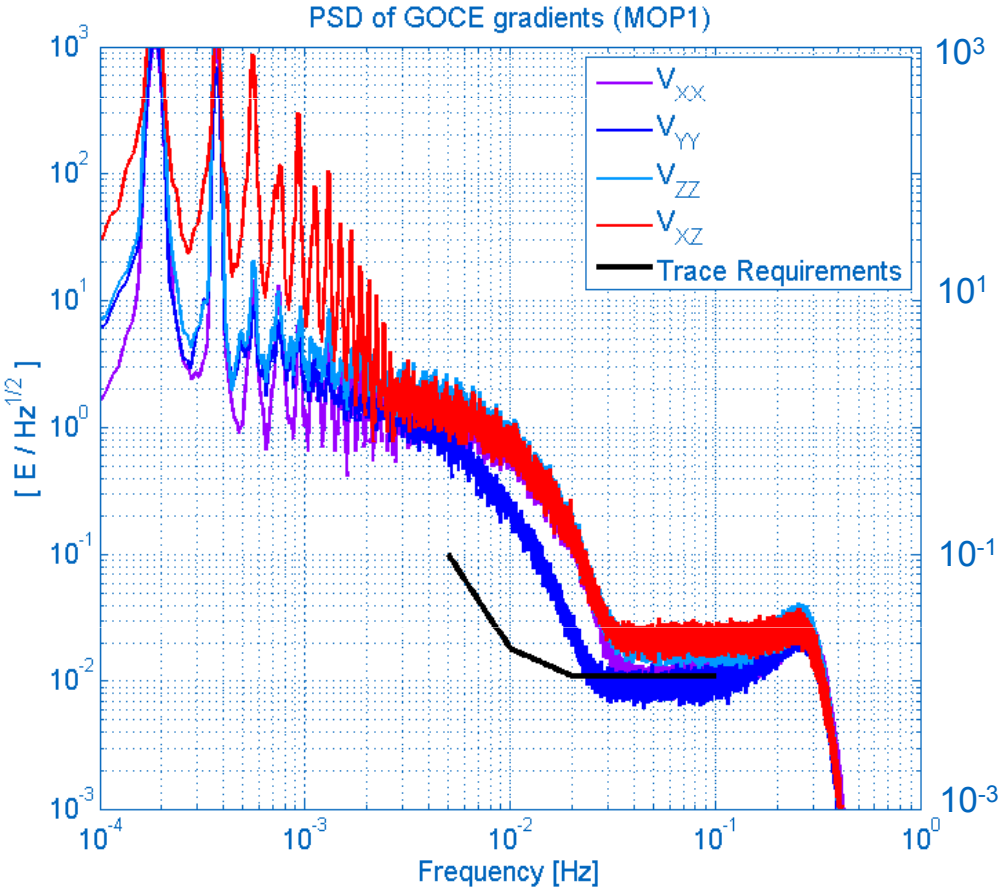


Motivation

- V_{xz} one of four good components
- Higher noise in lower frequencies

- Gravitational radial acceleration from GOCE?

$$\int V_{xz} dx \stackrel{?}{=} V_z$$



Contents

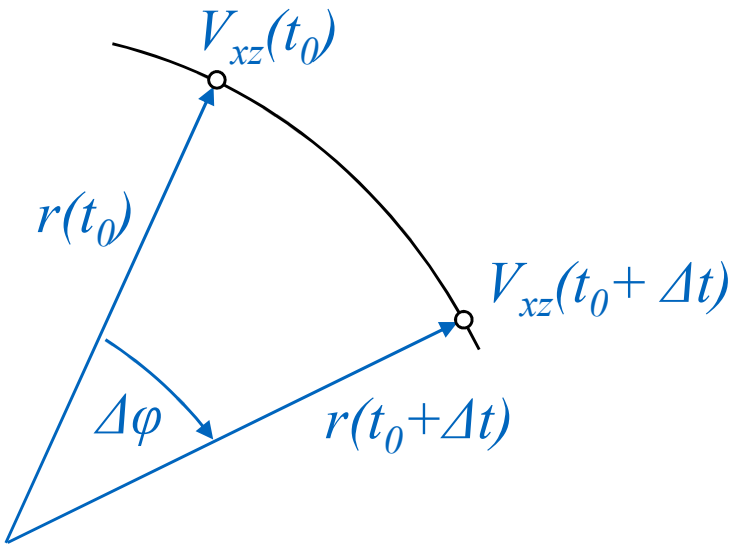
- Theory
  - Tensor theory
  - Numerical integration
- Test case:
  - Analytical proof
  - Test of the numerical integration
- Real world
  - Impact of GOCE‘s attitude
  - Results

Curvilinear coordinate system

- Covariant differentiation

$$V_{a;b} = V_{a,b} - \Gamma^{c}_{ab} V_c$$

$$V_{xz} = \frac{1}{r} V_{r\varphi} - \frac{1}{r^2} V_{\varphi}$$



- Integration along the orbit

$$\int V_{xz}\,dx = \int r\left(\frac{1}{r} V_{r\varphi} - \frac{1}{r^2} V_{\varphi}\right) d\varphi = V_r - \frac{1}{r} V \neq V_r$$

Test case

Circular orbit + only C_{20}

$$A = \frac{GM}{r^2}\left(\frac{R}{r}\right)^2 J_2$$

$$V_{xz} = 6 \cdot \frac{A}{r} \cdot \sin(2\varphi)$$

$$V_z - \frac{V}{r} = A \cdot (1 - 3\cos(2\varphi))$$

$$\int r \cdot V_{xz} d\varphi = -3A\cos(2\varphi) + k$$

Test case

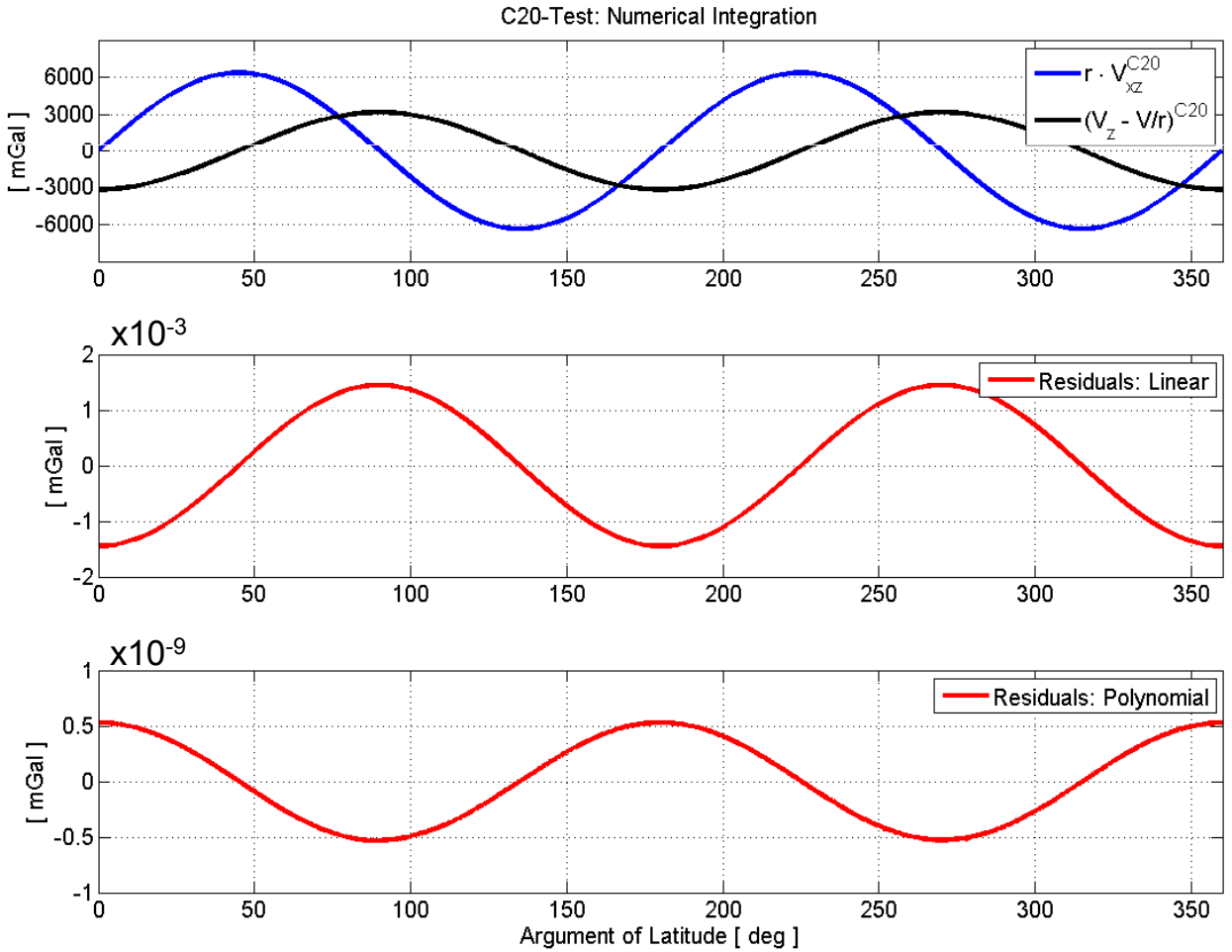
Circular orbit + only C_{20}

$$A = \frac{GM}{r^2}\left(\frac{R}{r}\right)^2 J_2$$

$$V_{xz} = 6 \cdot \frac{A}{r} \cdot \sin(2\varphi)$$

$$V_z - \frac{V}{r} = A \cdot (1 - 3\cos(2\varphi))$$

$$\int r \cdot V_{xz} d\varphi = -3A\cos(2\varphi) + k$$



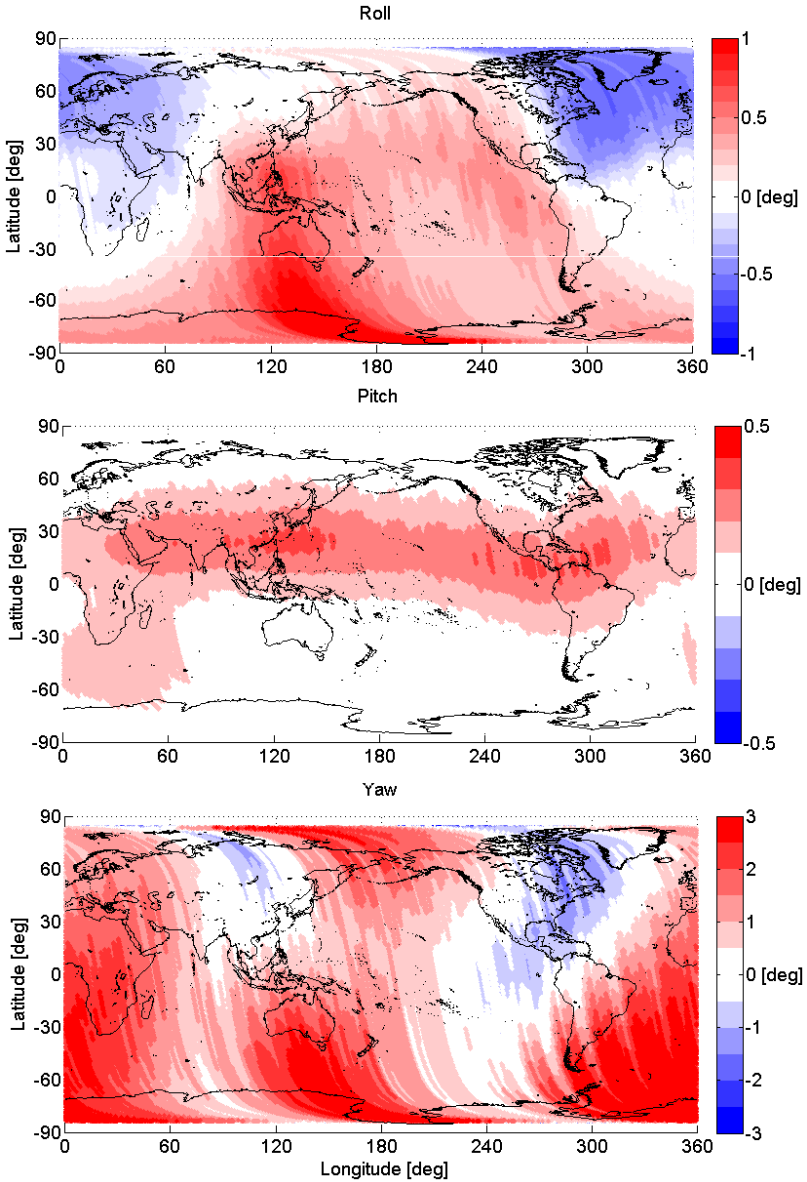
Real world

GOCE orbit + GOCE GG in GRF:

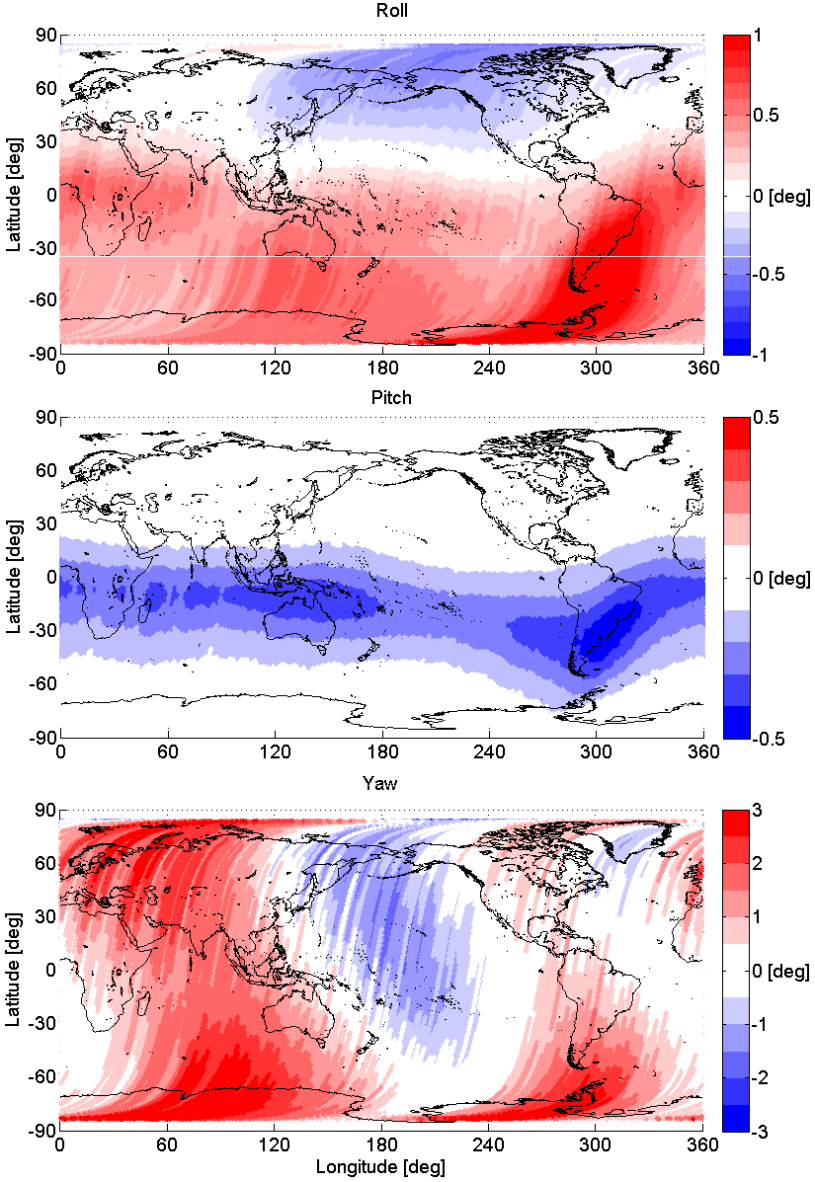
- Misalignment of GRF and LORF
- Varying altitude (~8 km)
- Noise

Attitude angles

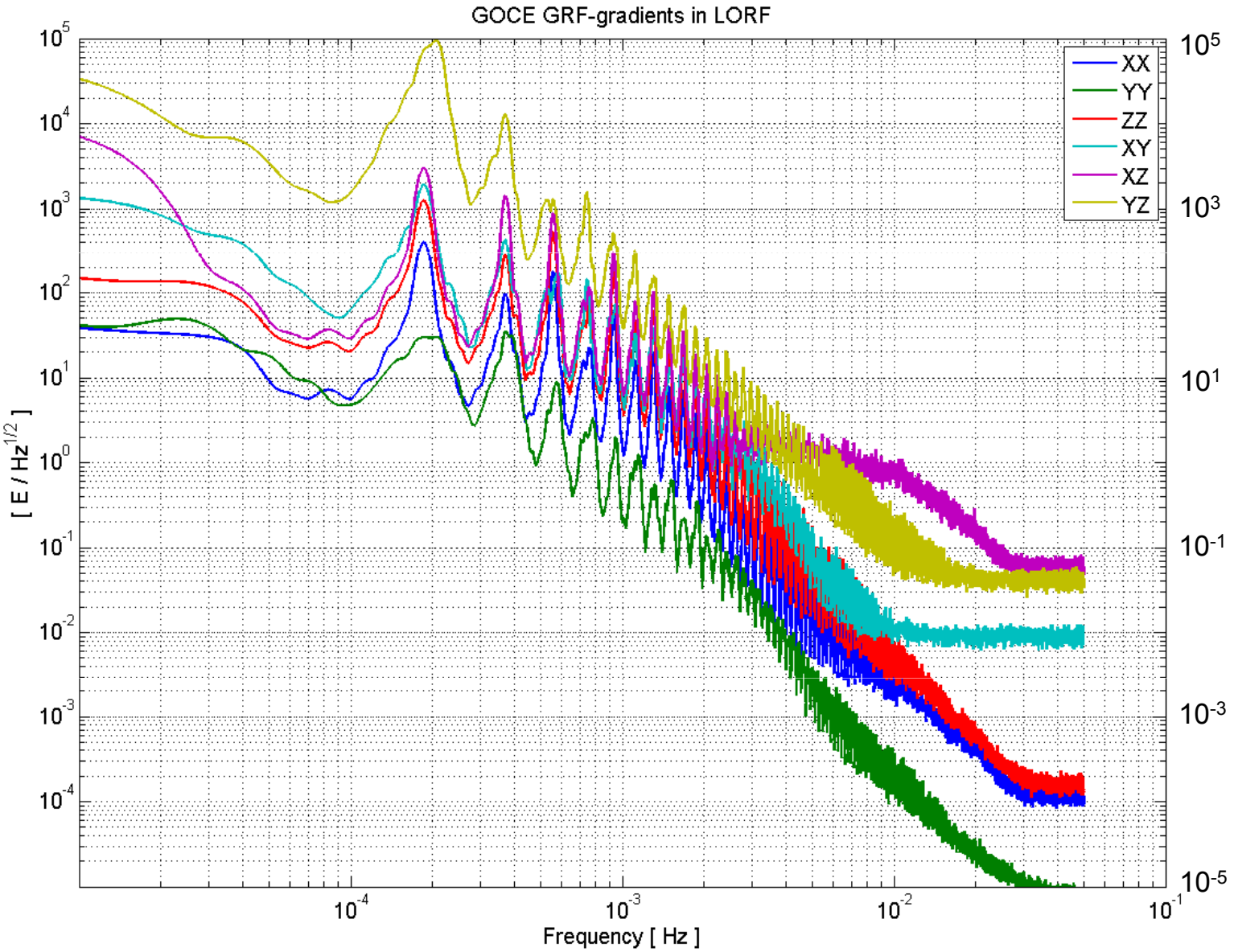
Ascending tracks



Descending tracks



Rotation: GRF → LORF

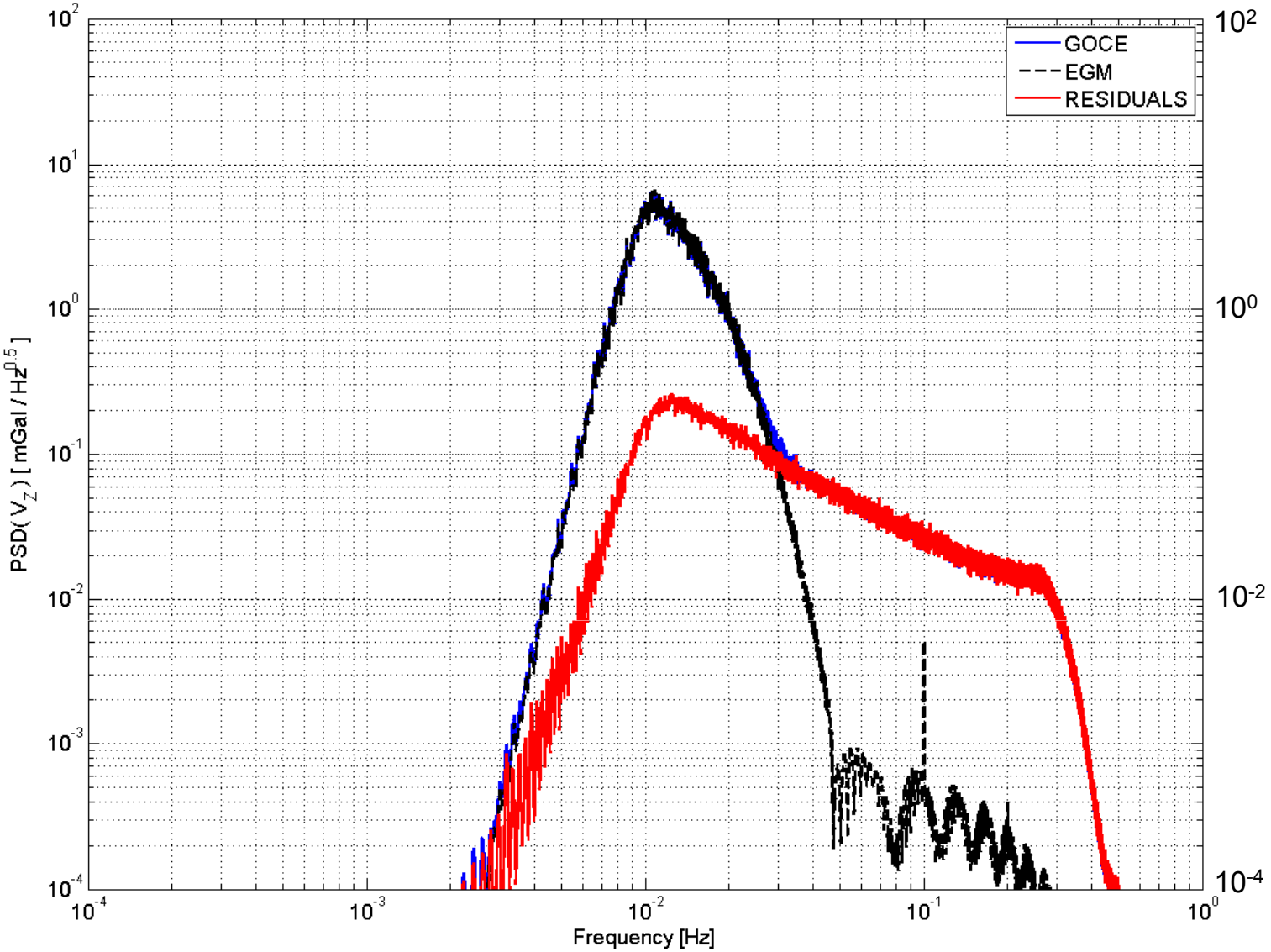


Real world

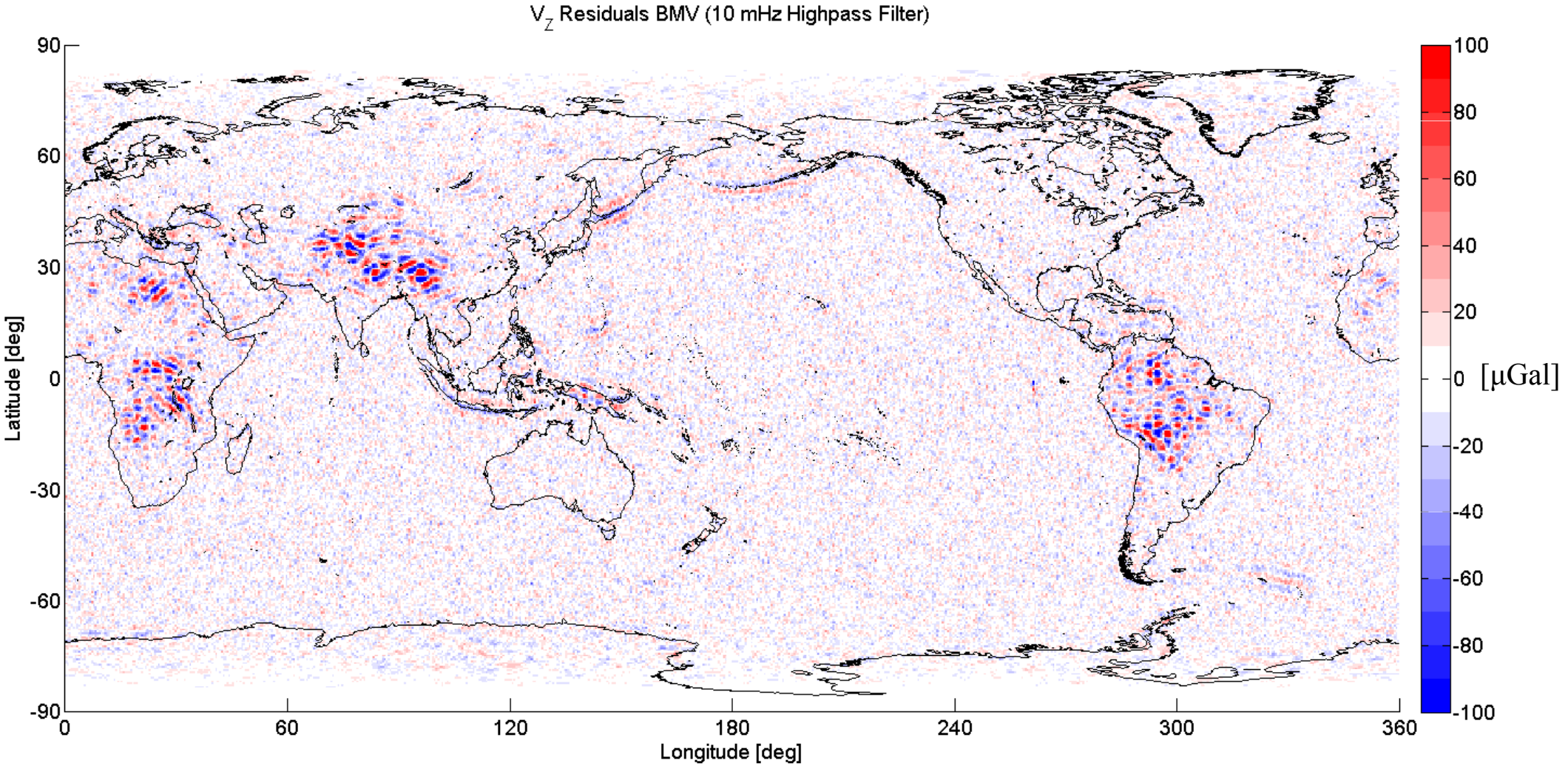
GOCE orbit + GOCE GG in GRF

- V_{xz} from GOCE (November + December 2009)
- V_z from EGM2008 up to d/o 250 at GOCE's positions
- Numerical integration with linear approximation
- Highpass filtered with a Butterworth (7th order) at 10 mHz
- No corrections of
  - Attitude misalignments
  - Varying altitude

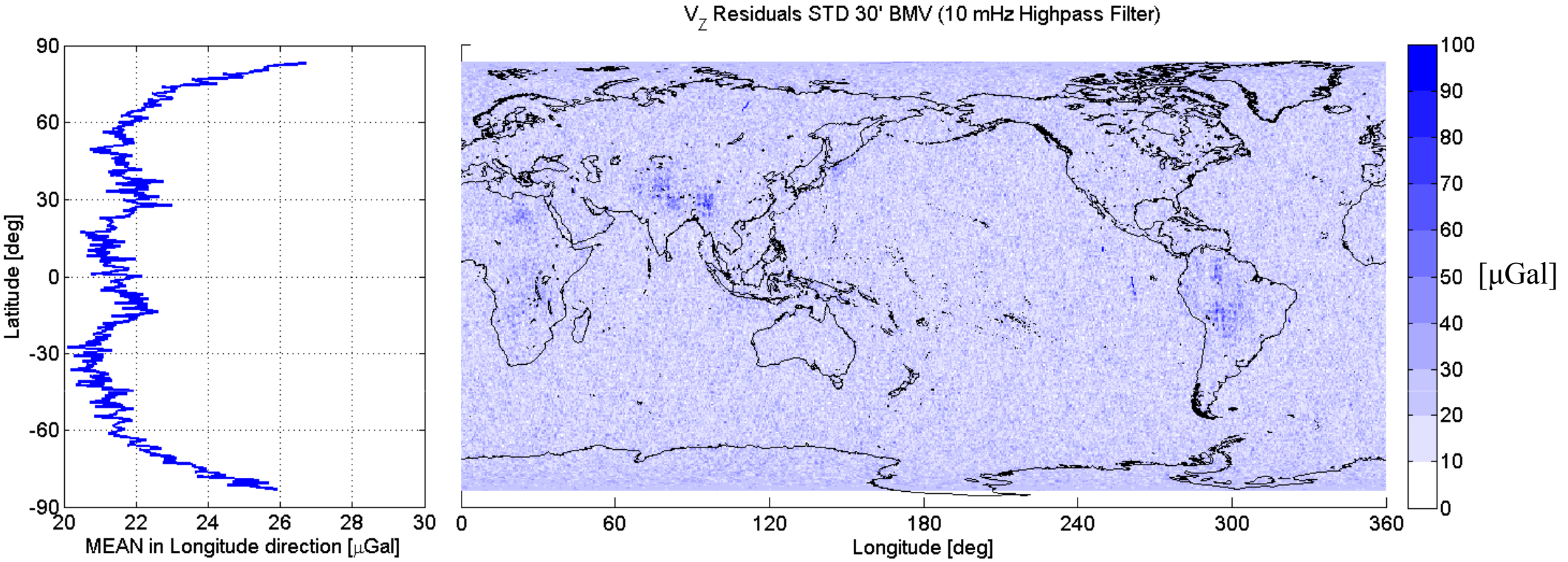
GOCE orbit + GOCE GG in GRF: spectral



GOCE GG in GRF: spatial (30‘ block mean values)



GOCE GG in GRF: spatial (30‘ block mean values) Standard deviations



Conclusion

- Integration of V_{xz} leads to V_z-V/r instead of V_z
- Impact of misalignment between GRF and LORF
- Difference of $LORF_v$ and $LORF_r$
- Impact of varying altitude