Application of Wavelet Support Vector Regression on SAR data Denoising

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Outline

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2. Support Vector Regression and Wavelet Kernel Function
3. SAR data Denoising based on WSVR
4. The experimental results and conclusion

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1. Introduction

➢ SVM & SVR

- **Support Vector Machine** (SVM)
  - a kind of machine learning theory based on mathematical statistics (Vapnik, 1996)
  - Advantages:
    Solving the machine learning problem of small sample;
    Improving generalization performance;
    Solving high-dimensional problem;
    Solving nonlinear problem;
    Avoiding the structure selection of neural network and local minimum point problem
  - SVM is used for classification and **regression.**
    Application:
    pattern recognition, computer vision, signal processing, function approximation, probability density estimate
- **Support Vector Regression** (SVR)
  - SVR method is introduced into SAR image filtering, by using of its restraining action for signal noise.

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2. Support Vector Regression and Wavelet Kernel Function

➢ Mathematical representations of Support Vector Regression

- Given training data set $T=\{(\mathbf{x}_1,y_1),(\mathbf{x}_2,y_2)\cdots(\mathbf{x}_N,y_N)\}$

$$\mathbf{x}_i \in R^m,\quad y_i \in R,\quad i=1,2,\cdots N$$

- considering the regression function

$$f(\mathbf{x}) = \mathbf{w}^T\varphi(\mathbf{x})+b$$

where $\mathbf{w}\in R^m$ is the weight vector, $b\in R$, $\varphi(\mathbf{x}): R^m \rightarrow R^{mh}$ mapping the input data into a higher dimensional feature space.

- To solve this regression problem, we minimize the object function:

$$\min_{\mathbf{w},b,e}\left[\frac{1}{2}\mathbf{w}^T\mathbf{w}+c\sum_{i=1}^{N}(\xi_i+\xi_i^*)\right]$$

$$s.t.\quad \begin{cases} y_i-\mathbf{w}^T\varphi(\mathbf{x}_i)-b\le\varepsilon+\xi_i \\ \mathbf{w}^T\varphi(\mathbf{x}_i)+b-y_i\le\varepsilon+\xi_i^* \quad i=1,\cdots,N \\ \xi_i,\ \xi_i^*>0 \end{cases}$$

ε --- insensitive threshold; when controlling the training error in an insensitive interval $[-\varepsilon,+\varepsilon]$, we think that there is no loss of training error.

ξ^* *and* ξ --- are slack variables; The second part of above formula is training loss.

C --- positive constant, which used to keep the balance of smoothing extent between training loss and fitting result.

Mathematical representations of Support Vector Regression (2)

- From this we can establish corresponding Lagrange functional and optimize it. The solution is given by:

$$w = \sum_{i=1}^{N} (\alpha_i^* - \alpha_i) \varphi(x_i)$$

α_i^* and α_i are Lagrange multipliers. The result of function fitting could be represented as:

$$f(x) = \sum_{i=1}^{N} (\alpha_i^* - \alpha_i) K(x_i, x) + b$$

$K(x_i, x)$ is the kernel function, from which we can directly calculate the inner product in the higher dimensional feature space, that is

$$K(x_i, x_j) = \varphi(x_i)^T \varphi(x_j)$$

- Form the feature of Kernel functions: implicitly map low-dimensional space to higher-dimensional linear decomposable space, and compute the inner product in higher-dimensional space. it is the realizable guarantee of SVR method.
- The optimized problem to solve SVR is a convex quadratic programming problem. It is a global optimal solution instead of a local optimal solution. In this respect, it is better than neural network method.

➢ Wavelet Support Vector Regression

- On the basis of Mercer Condition, the wavelet function can be constructed, and then the Wavelet Support Vector Regression can be obtained.
- Here we consider using Morlet mother wavelet to construct the wavelet kernel function :

$$K_w(x - x') = \prod_{i=1}^{d} \left[\frac{\cos 1.75(x_i - x_i')}{a} \right] \exp\left[-\frac{(x_i - x_i')^2}{2a^2} \right]$$

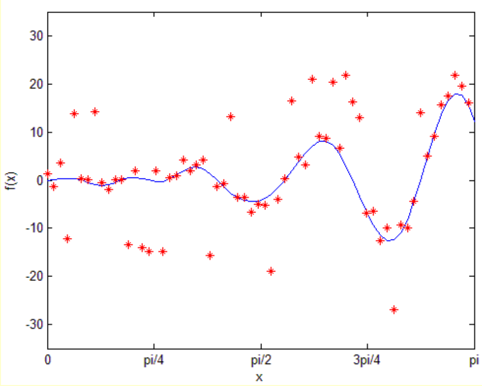
- The advantage of wavelet kernel function lies that wavelet has better expressive ability for the signal detail , and for arbitrary complex function, it can obtain better fitting result than any traditional kernel

3. SAR data Denoising based on WSVR

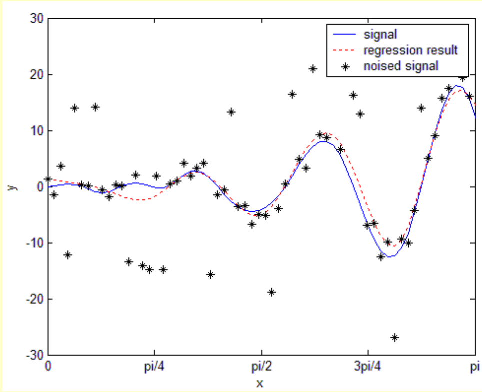
- **The feasibility of WSVR using in the signal filtering**
- The experiment of 1-dimension signal regression with WSVR

$$f(x) = x\sin(4\pi x)\exp(-x^2+1) + (2x^2)\tan(10x)\cos(2\pi x)$$

where x denotes scalar input, In this case, the SVR parameter $\varepsilon = 0.3$, $c = 0.5$ and use the wavelet function as the SVR kernel, where the scale parameter $a = 0.8$



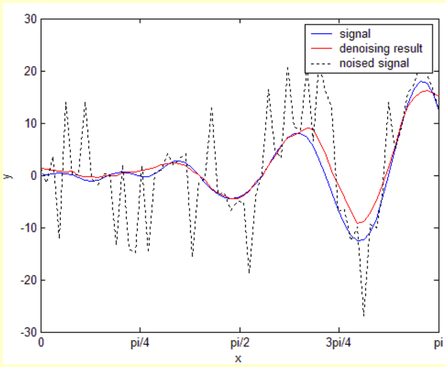
1-D original signal (blue curve) and the contaminated noise (red asterisks)



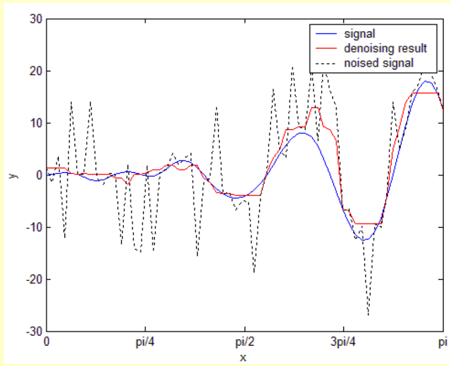
WSVR Results (black asterisks are sample noisy)

Filtering Result evaluation for one-dimensional signal

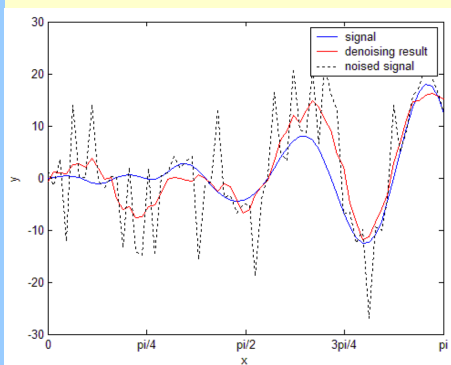
$$SNR = 10\times\log_{10}\left[\sum_{x=1}^{N}\frac{f(x)^2}{\left(\hat{f}(x)-f(x)\right)^2}\right]$$



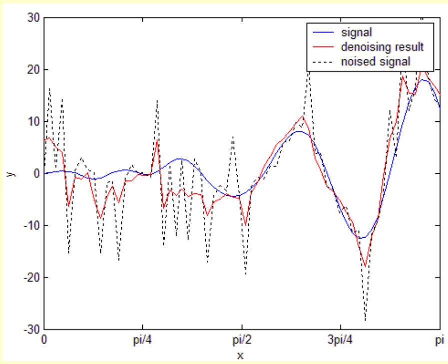
WSVR,SNR = 51.7361



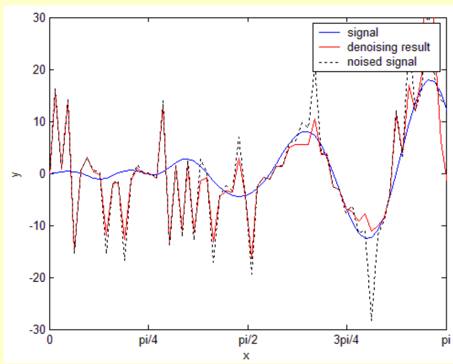
Median, SNR = 51.2782



Gauss, SNR = 44.1924



Wavelet Soft, SNR = 36.2937



Lee, SNR = 38.9392

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➢ the noise model of intensity imagery for SAR

- The model of SAR
  - the coherent speckle noises in SAR is multiplicative, the model is:

$$I = \sigma n$$

  - where I is the observed value of image intensity, n is multiplicative random noise, σ is the true image.
- The model of Logarithmic SAR image
  - Logarithmical transformation changes the multiplicative noise into additive noise.
  - the noise model of logarithmical transformed SAR image

$$I_L = \sigma_L + n_G + n_P$$

  - where I_L is the observed value of logarithmical image, σ_L is the truth value of logarithmical image, n_G and n_P are separately gauss noise and salt-and-pepper noise.

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➢ Classical denoising methods for SAR

- **adaptive filtering based on local statistical property** (Lee, 1986)
  - assume a type of noise firstly, and then to do filtering operation based on the distribution characters. The representative filter are Frost filter, Lee filter, Kuan filter and GMAP filter .
- **wavelet method** (Guo et al., 1994; Donoho, 1995)
  - wavelet's soft-threshold value de-noising method
  - statistical modeling based on wavelet coefficients
- **the filtering based on Markov random field** (Geman et.al., 1984),
  - assume the image prior probability using Markov random field, then solve the issue in the frame of maximum posterior probability estimation based on Bayesian theorem.

SAR imagery filtering based on WSVR

- SAR imagery is a kind of two-dimensional signal, the training data set of the image is:

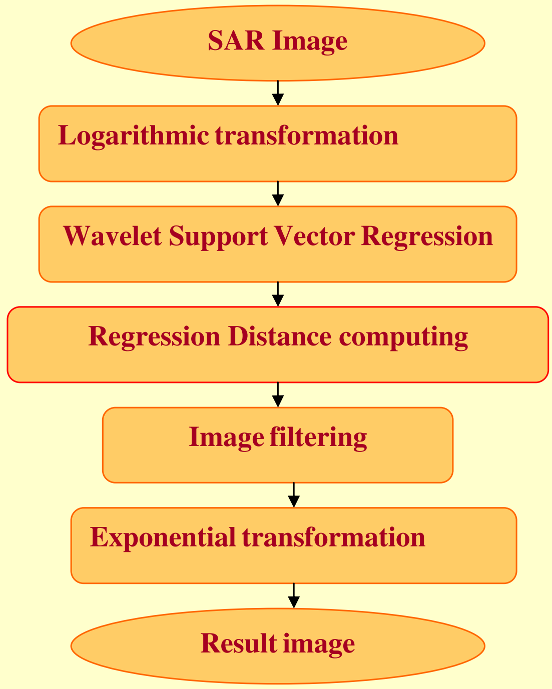
$$T_{image} = \{(\mathbf{pos}_1, gray_1), (\mathbf{pos}_2, gray_2) \cdots (\mathbf{pos}_{N\times M}, gray_{N\times M})\}$$

- The result of function fitting could be represented as:

$$f_{ima}(\mathbf{x}) = \sum_{i=1}^{M}\sum_{j=1}^{N}(\alpha_{ij}^{*} - \alpha_{ij}) K_w(\mathbf{x}_{ij}, \mathbf{x}) + b$$

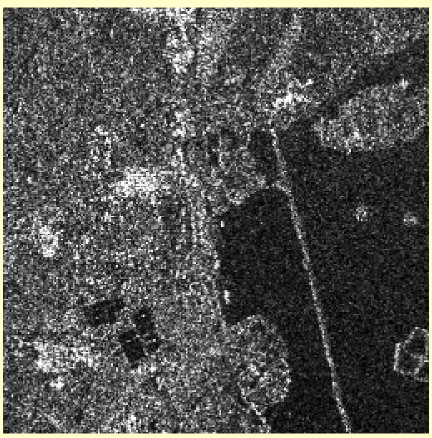
- The algorithm flow chart of WSVR filtering for SAR
- --- Logarithmical transformation changes the
- multiplicative noise of image into additive noise
- --- the regression distance is used as judgment index
- of the noise type: $D_{reg} = |\hat{f} - f|$

$$\hat{I} = \begin{cases} \hat{I}_{median} & D_{reg} > T_D \\ \hat{I}_{gauss} & D_{reg} \le T_D \end{cases}$$

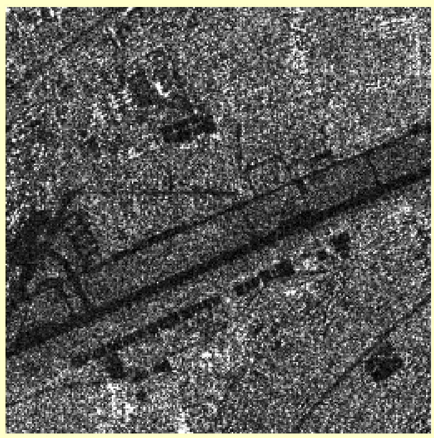


4. The experimental results and conclusions

Original SAR image

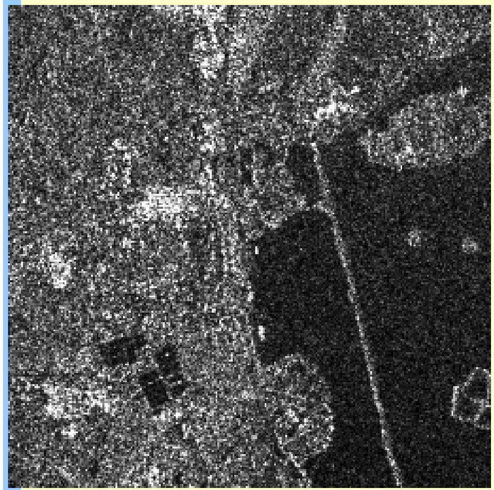


(a) the areas of water and land common boundary

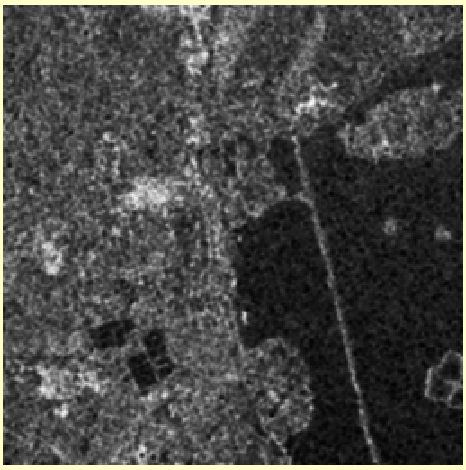


(b) airport area

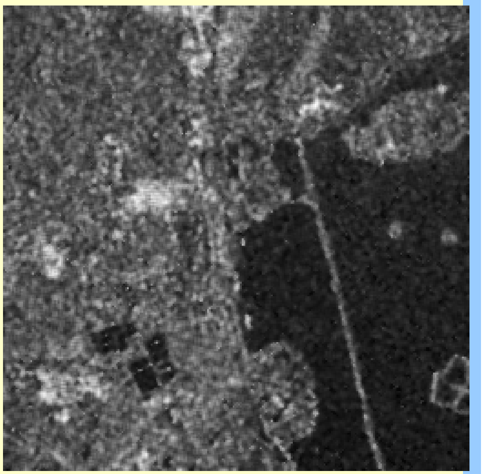
Here we have chosen two typical areas in SAR image to proceed the experiment, which are the areas of water and land common boundary and airport . The size of two image are all of 256×256.



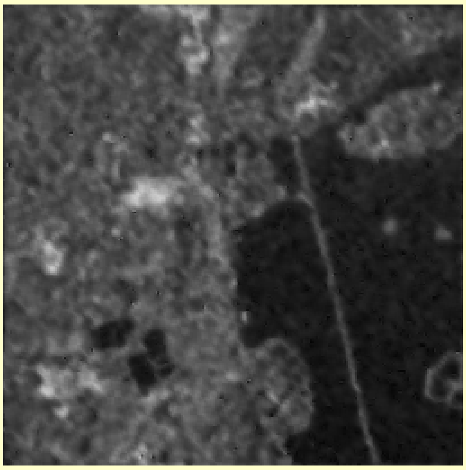
Original SAR image



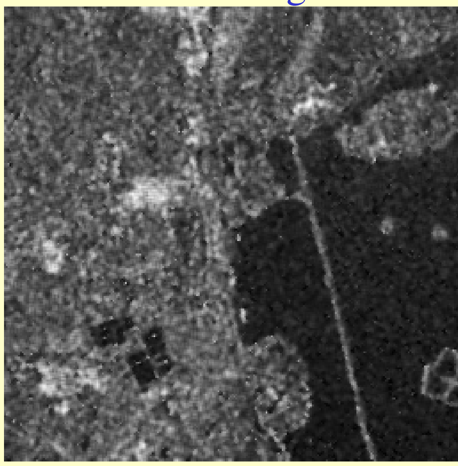
WSVR filtering



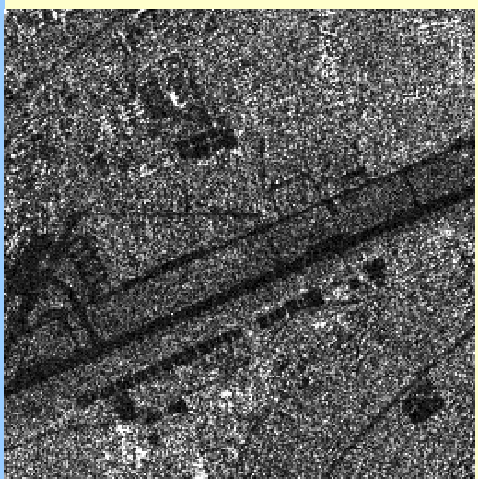
Kuan filtering



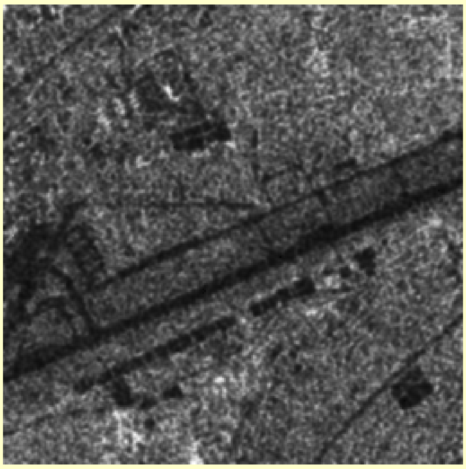
wavelet soft threshold value filtering



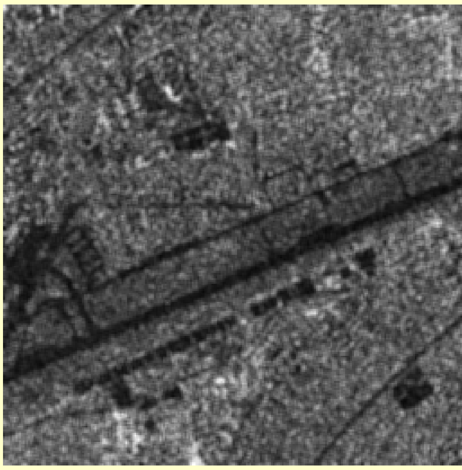
LEE filtering



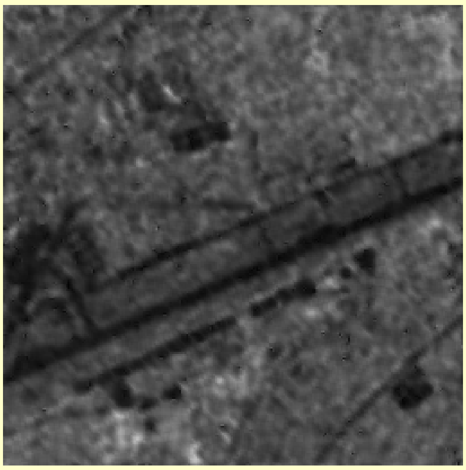
Original SAR image



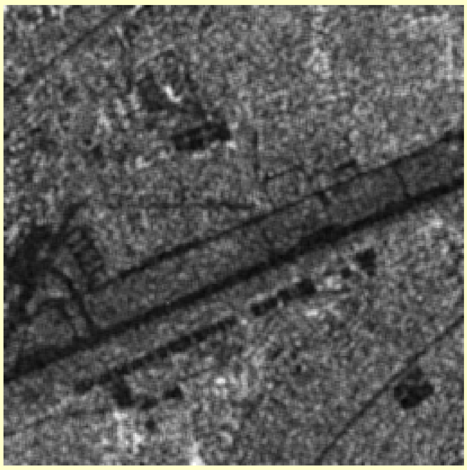
WSVR filtering



Kuan filtering



wavelet soft threshold value filtering



LEE filtering

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Result evaluation of SAR data denoising

- In this paper, we choose four quantized indicators to evaluate the filtering result, which including the gain of *ENL* G_{ENL}, the gain of *STD* G_{STD}, edge enhancement index *EEI* and the mean of ratio *ER*.
- Evaluation index statistics of SAR image filtering results in the area of Fig (a)

Filtering method	G_{ENL}	G_{STD}	EEI	ER
WSVR	9.7853	0.3385	0.2324	1.2152
Lee	6.9773	0.4068	0.2039	1.3378
Kuan	9.2088	0.3582	0.1851	1.3529
Wavelet soft threshold filtering	5.5992	0.3484	0.1219	1.1628

- Evaluation index statistics of SAR image filtering results in the area of Fig (b)

Filtering method	G_{ENL}	G_{STD}	EEI	ER
WSVR	9.4924	0.2880	0.2182	1.2149
Lee	11.6879	0.2887	0.2051	1.3455
Kuan	13.6881	0.2670	0.1867	1.3592
Wavelet soft threshold filtering	9.1943	0.2680	0.1253	1.1581

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Conclusion

- WSVR could well reduce the additive random noise and multiplicative salt-and-pepper noise for SAR image filtering.
- Comparing to other image denoising method, such as KUAN filtering, enhanced LEE filtering and the wavelet soft threshold filtering, WSVR filtering could keep edge feature better.
- But WSVR took a longer time. For real-time demanding applications, we need to research the method of high-speed realization.

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- ***Thanks for your attention !***