On the determination of the optimal α in the uniform Tykhonov-Phillips regularization (α-weighted BLE)

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Numerical tests have documented that the estimate of type BLUUE of the parameter vector within a linear *Gauss-Markov model* is *not* robust against *outliers* in the stochastic observation vector. It is for this reason that *we give up* the postulate of unbiasedness, but keeping the set-up of a *linear estimation* of homogeneous type. Ever since *Tykhonov* (1963) and *Phillips* (1962) introduced the *hybrid minimum norm approximation solution* (HAPS) of a *linear improperly posed problem* there has been left the open problem to evaluate the weighting factor α between the least-squares norm and the minimum length norm of the unknown parameters. In most applications of *Tykhonov-Phillips* type of regularization the weighting factor α is determined by simulation studies, such as by means of *L*-Curve (*Hansen* 1992) or the C_p-Plot (*Mallows* 1973), but according to the literature listed below also optimization techniques have been applied. Here we aim at an objective method to determine the *weighting factor* α within α-HAPS. According to Grafarend and Schaffrin (1993), updated by Schaffrin (2000), the best linear estimation of type α-homBLE (*α-weighted Best homogeneously Linear Estimation*) which is based on *hybrid norm optimization* of type (i) minimum variance *and* (ii) minimum bias, is *equivalent to* α-HAPS under the condition that if we choose the weight matrix in the least squares norm as the inverse matrix of the variance covariance matrix of the observations *as well as* the weight matrix in the minimum norm acting on the unknown parameter vector as the inverse substitute bias weight matrix. We have developed a new method of determining the optimal regularization parameter α in uniform Tykhonov-Phillips regularization (α-weighted BLE) by minimizing the trace of the Mean Square Error matrix MSE (A-optimal design) in the general case. This estimation formula is closed, which provides us not only with the optimal regularization parameter but also with more quicker and more practical solutions than by the simulation methods. Further, it has been shown that the optimal ridge parameter *k* in *ridge regression* as developed by *Hoerl and Kennard* (1970a, 1970b) and *Hoerl, Kennard and Baldwin* (1975) is just the special case of our general solution by A-optimal design. In a case study, both model and estimators are tested and analyzed with numerical results.